

On strangeness production in the reactions $pp \rightarrow p\Lambda^0 K^+$ and $pn \rightarrow n\Lambda^0 K^+$ near threshold

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February 9, 2008

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Abstract

The cross sections for the reactions $pp \rightarrow p\Lambda^0 K^+$ and $pn \rightarrow n\Lambda^0 K^+$ are calculated near threshold of the final states. The theoretical ratio of the cross sections $R = \sigma(pn \rightarrow n\Lambda^0 K^+)/\sigma(pp \rightarrow p\Lambda^0 K^+) \simeq 3$ shows the enhancement of the pn interaction with respect to the pp interaction near threshold of the strangeness production $N\Lambda^0 K^+$. Such an enhancement is caused by the contribution of the np interaction in the isospin-singlet state, which is stronger than the pn interaction in the isospin-triplet state. For the confirmation of this result we calculate the cross sections for the reactions $pp \rightarrow pp\pi^0$, $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$ near threshold of the final states. The theoretical cross sections agree well with the experimental data.

PACS: 11.10.Ef, 12.39. Fe, 13.75.Jz, 14.20.Jn

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1 Introduction

Recent experiments on the production of K^+ – mesons in the reaction $pd \rightarrow K^+ X N$, where X is a hadronic state with strangeness $S = -1$ and baryon number $B = 2$ and N is a nucleon, have led to the prediction for the ratio of the cross sections for the reactions $pp \rightarrow K^+ X$ and $pn \rightarrow K^+ X$

$$R = \frac{\sigma(pn \rightarrow K^+ X)}{\sigma(pp \rightarrow K^+ X)} \sim 3 - 4. \quad (1.1)$$

The estimate has been obtained for proton beam energies $T_p = 1.83 \text{ GeV}$ and $T_p = 2.02 \text{ GeV}$ giving $R \sim 3$ and $R \sim 4$, respectively, to fit the experimental data on the differential cross section for the reaction $pd \rightarrow K^+ X N$ [1]. The parameter R has been used as an input parameter. This has been justified by the fact that the scattering of the proton by the deuteron can be treated in the *impulse* approximation [2], since there is no indication for any collective target behaviour [1] (see also [3]).

Since the main contribution to the cross sections for the reactions $pp \rightarrow K^+ X$ and $pn \rightarrow K^+ X$ comes from the channels $pp \rightarrow p\Lambda^0 K^+$ and $pn \rightarrow n\Lambda^0 K^+$ [1, 4], the ratio Eq.(1.1) can be rewritten as follows

$$R = \frac{\sigma(pn \rightarrow n\Lambda^0 K^+)}{\sigma(pp \rightarrow p\Lambda^0 K^+)} \sim 3 - 4. \quad (1.2)$$

Can such a ratio be valid near threshold of the final state $N\Lambda^0 K^+$? [5].

A theoretical explanation of the ratio $R \sim 3 - 4$ near threshold of the final $N\Lambda^0 K^+$ state has been proposed in [4] and [6]. As has been pointed out in [6], the value $R \sim 3$ can be mainly due to the dominant contribution of the ρ –meson exchange in addition to the contribution of the resonances $N(1535)$ and $N(1650)$ with the quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ [7], described within the Breit–Wigner approach [8]. In [4] the cross sections for the reactions $pN \rightarrow N\Lambda^0 K^+$ have been calculated within the resonance model of NN scattering for energies of the incident proton far from threshold. Within such an approach the ratio R can be obtained assuming the validity of the cross sections in the vicinity of threshold [9]. Recently [10] the strangeness production in pp collisions $pp \rightarrow p\Lambda^0 K^+$ and $pp \rightarrow p\Sigma^0 K^+$ near threshold of the final states has been analysed by using chiral Lagrangians with linear realization of chiral symmetry and non–derivative meson–baryon couplings [11]–[14]. The matrix elements of the transitions $pp \rightarrow p\Lambda^0 K^+$ and $pp \rightarrow p\Sigma^0 K^+$ have been calculated in the one–meson exchange approximation [15]. For the calculation of the amplitudes of the reactions $pp \rightarrow p\Lambda^0 K^+$ and $pp \rightarrow p\Sigma^0 K^+$ there has been taken into account the pp rescattering in the initial state, i.e. the initial state interaction [16], and $p\Lambda^0 K^+$ and $p\Sigma^0 K^+$ final–state interaction [17, 18]. As has been shown in [19] the same analysis of the strangeness production has turned out to be very useful for the correct description of the reactions $K^- d \rightarrow NY$ of $K^- d$ scattering at threshold, where $NY = p\Sigma^-, n\Sigma^0$ and $n\Lambda^0$. In this paper we apply such an analysis of the strangeness production to the calculation of the cross sections for the reactions $pN \rightarrow N\Lambda^0 K^+$ near threshold of the final state $N\Lambda^0 K^+$ and the ratio R .

The paper is organised as follows. In Section 2 we calculate the cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$. The obtained result agrees with the experimental data [20] with an accuracy better 16 %. We show that the contribution of the vector ρ – and ω^0 –meson

exchanges to the cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ makes up about 9% only. In Section 3 we calculate the cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$. We calculate the cross sections for the reactions with the pn pair in the state with isospin $I = 1$ and isospin $I = 0$ and demonstrate that $R^{(I=1)} = \sigma((pn)_{I=1} \rightarrow n\Lambda^0 K^+)/\sigma(pp \rightarrow p\Lambda^0 K^+) \simeq 1$. We find that the contribution of the vector ρ - and ω -meson exchanges to the cross section for the reaction $(pn)_{I=1} \rightarrow n\Lambda^0 K^+$ with the pn pair in the isospin-triplet state is about 3%. In turn, the vector-meson exchanges do not influence on the value of the cross section for the reaction $(pn)_{I=0} \rightarrow n\Lambda^0 K^+$ with the pn pair in the isospin-singlet state. In Section 4 we calculate the ratio R of the cross sections for the reactions $pn \rightarrow n\Lambda^0 K^+$ and $pp \rightarrow p\Lambda^0 K^+$. We get $R \simeq 3$. This agrees with the enhancement of the pn interaction with respect to the pp interaction in the reactions of the strangeness production observed by the ANKE-Collaboration at COSY and the theoretical prediction in [4] and [6]. We show that such an enhancement is caused by the contribution of pn interaction in the isospin-singlet state $(pn)_{I=0}$, which is stronger than the pn interaction in the isospin-triplet state $(pn)_{I=1}$. In Section 5 we analyse the contribution of the resonances $N(1535)$ and $N(1650)$, treating them as elementary particles [21]–[27]. We show that the contribution of these resonances to the amplitudes of the reactions $pN \rightarrow N\Lambda^0 K^+$ can be neglected with respect to the contribution of the ground state baryon octet coupled to the octets of the pseudoscalar and scalar mesons. In Section 6 we calculate the cross section for the reaction $pp \rightarrow pp\pi^0$, caused by the pure ${}^3P_0 \rightarrow {}^1S_0$ transition with pseudoscalar and scalar meson exchanges. Near threshold the theoretical cross section agrees well with the experimental data. In Section 7 we calculate the cross sections for the reactions $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$. We show that the theoretical cross section for the reaction $\pi^- p \rightarrow \Lambda^0 K^0$ agrees well with the experimental data for the energy excess $\varepsilon \leq 13$ MeV [28] and the results obtained within $SU(3)$ chiral dynamics with coupled channels [29]. We show that for the correct description of the cross sections for the reactions $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$ the contribution of the resonances $N(1535)$ and $N(1650)$ as well as $\Sigma(1750)$ is very important due to the cancellation of the non-resonance contribution. The dominant role of resonances $N(1535)$ and $N(1650)$ in the amplitudes of the reactions $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$ has been pointed out by Kaiser *et al.* [29]. In the Conclusion we discuss the obtained results. In Appendix A we analyse the contribution of the final-state interaction to the amplitudes of the reactions $pN \rightarrow N\Lambda^0 K^+$.

2 Cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ near threshold

Following [4, 6] and [10, 15, 19] we define the amplitude of the transition $pp \rightarrow p\Lambda^0 K^+$ in the tree-approximation with one-meson exchanges. A complete set of Feynman diagrams, describing the transition $pp \rightarrow p\Lambda^0 K^+$ near threshold of the final $p\Lambda^0 K^+$ state, is depicted in Fig.1 and Fig.2. The calculation of these diagrams we carry out using chiral Lagrangian with non-derivative meson-baryon couplings invariant under linear transformations of chiral $SU(3) \times SU(3)$ symmetry [11]–[14]. Since masses of scalar partners of pseudoscalar mesons are not well-defined, we calculate the amplitudes in the limit of infinite masses of scalar mesons. According to [12, 13], such a limit corresponds to the calculation of the amplitudes with chiral Lagrangian invariant under non-linear transformations of chiral

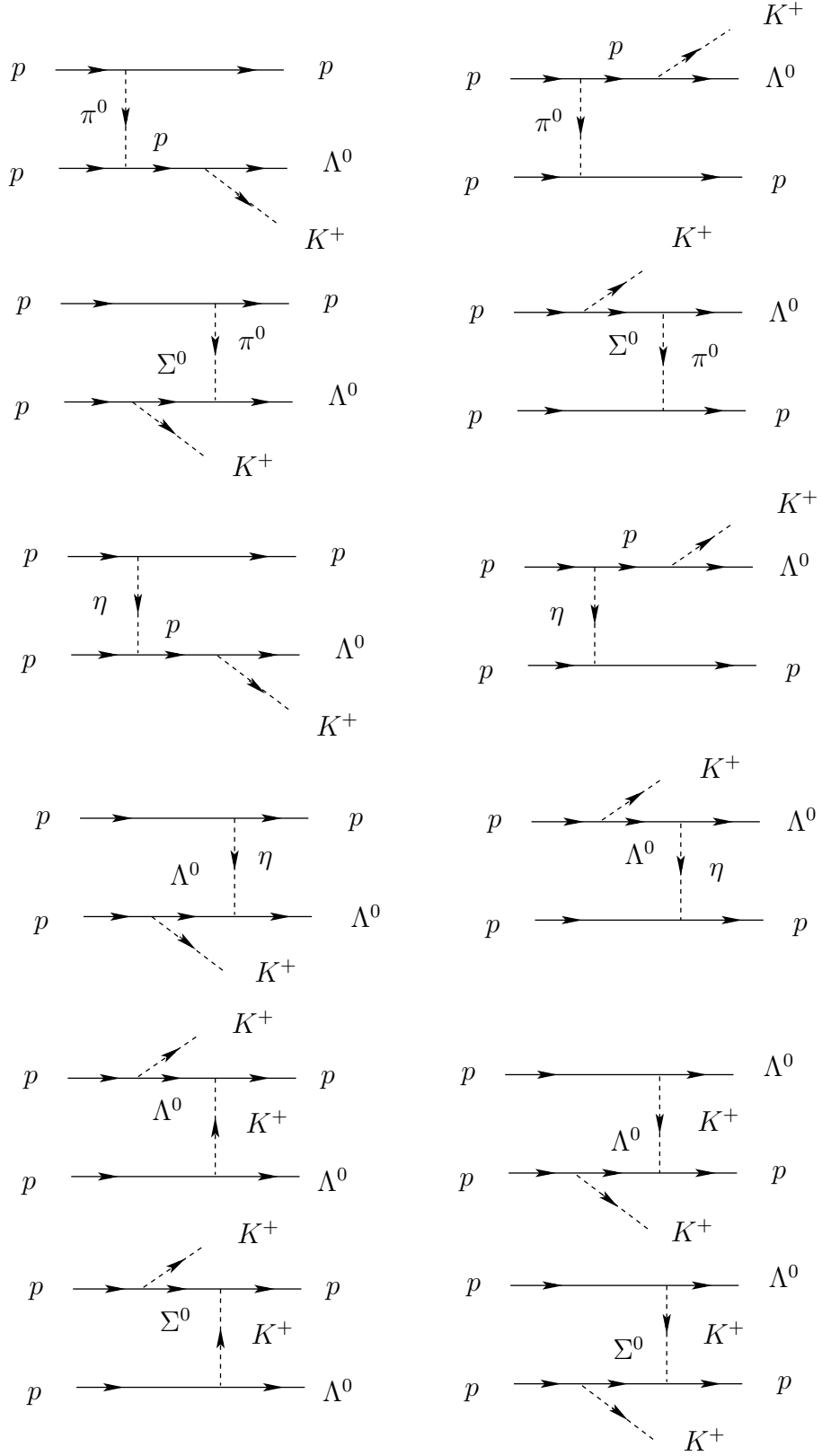


Figure 1: The set of Feynman diagrams contributing to the effective coupling constant $C_{p\Lambda^0 K^+}^{pp}$ of the $pp \rightarrow p\Lambda^0 K^+$ transition in the one-pseudoscalar meson exchange approximation.

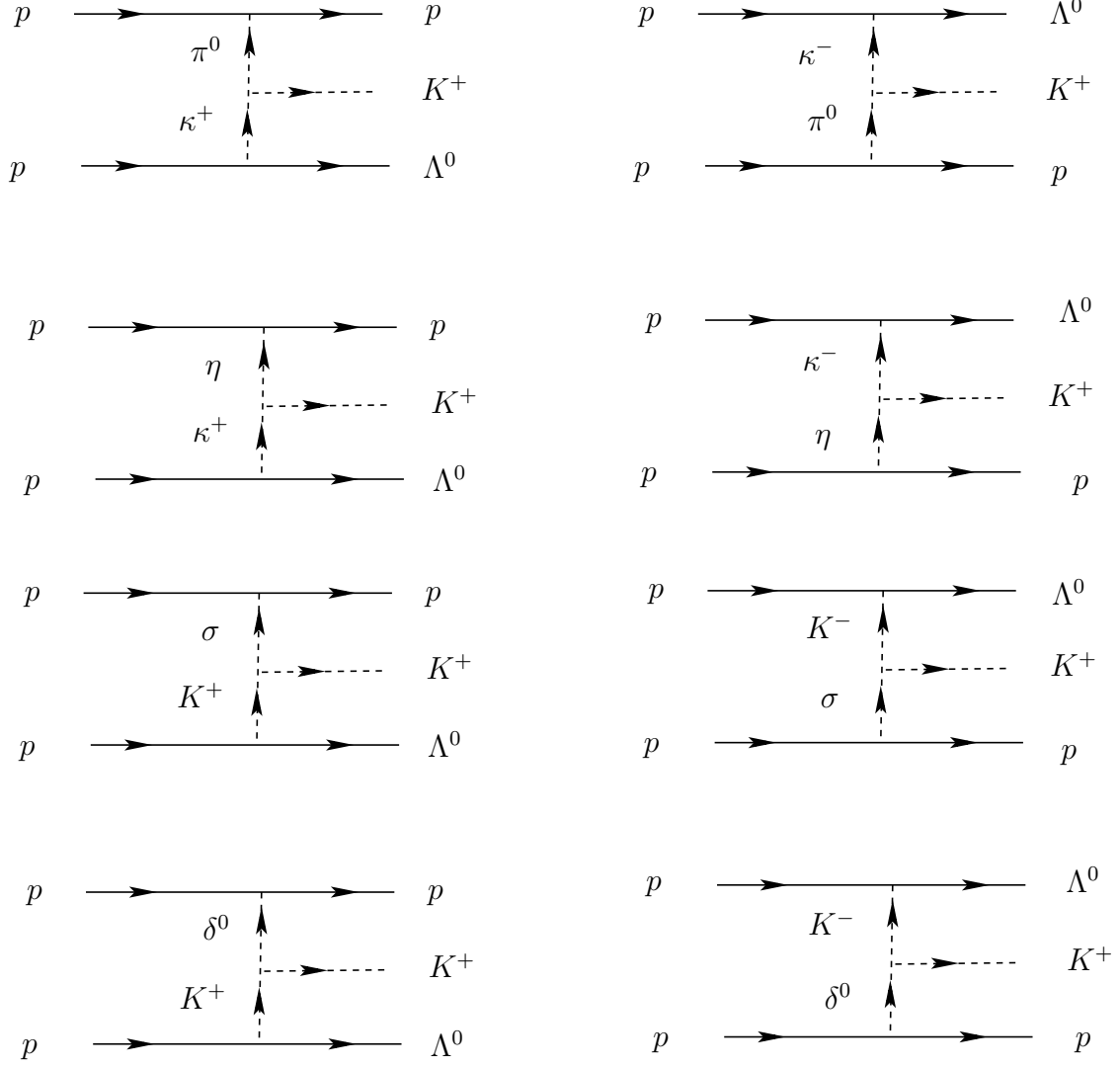


Figure 2: The set of Feynman diagrams contributing to the effective coupling constant $C_{p\Lambda^0 K^+}^{pp}$ of the $pp \rightarrow p\Lambda^0 K^+$ transition in the one-scalar meson exchange approximation.

$SU(3) \times SU(3)$ symmetry[14]. Of course, such an equivalence has been proved only in the tree-approximation [12]–[14].

For the derivation of the analytical expressions of these diagrams we use the meson–baryon interactions (see Eqs.(D.4), (D.7) and (D.9) of Ref.[19] and also [30]) and the wave functions of the initial and the final states in the particle number representation:

$$\begin{aligned} |p(\vec{p}_1, \sigma_1)p(\vec{p}_2, \sigma_2)\rangle &= a_p^\dagger(\vec{p}_1, \sigma_1)a_p^\dagger(\vec{p}_2, \sigma_2)|0\rangle, \\ |p(\vec{k}_p, \sigma_p)\Lambda^0(\vec{k}_\Lambda, \sigma_\Lambda)K^+(\vec{Q}_K)\rangle &= a_p^\dagger(\vec{k}_p, \sigma_p)a_\Lambda^\dagger(\vec{k}_\Lambda, \sigma_\Lambda)c_{K^+}^\dagger(\vec{Q}_K)|0\rangle, \end{aligned} \quad (2.1)$$

where creation (annihilation) operators of baryons and the K^- –meson obey standard relativistic covariant anti-commutation and commutation relations [19].

Near threshold of the final state $p\Lambda^0 K^+$ the Feynman diagrams in Fig.1 and Fig.2 can be described by the effective local Lagrangian of the transition $pp \rightarrow p\Lambda^0 K^+$ [10, 19]:

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{pp \rightarrow p\Lambda^0 K^+}(x) &= A_{p\Lambda^0 K^+}^{pp} [\bar{\Lambda}^0(x)p(x)][\bar{p}(x)i\gamma^5 p(x)] K^{+\dagger}(x) \\ &+ B_{p\Lambda^0 K^+}^{pp} [\bar{\Lambda}^0(x)i\gamma^5 p(x)][\bar{p}(x)p(x)] K^{+\dagger}(x), \end{aligned} \quad (2.2)$$

where the coefficients $A_{p\Lambda^0 K^+}^{pp}$ and $B_{p\Lambda^0 K^+}^{pp}$ are equal to

$$\begin{aligned} A_{p\Lambda^0 K^+}^{pp} &= -\frac{1}{\sqrt{3}} \frac{(3-2\alpha_D)g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\ &+ \frac{2}{\sqrt{3}} \frac{\alpha_D(2\alpha_D-1)g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} \\ &- \frac{1}{3\sqrt{3}} \frac{(3-2\alpha_D)(3-4\alpha_D)^2 g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\ &+ \frac{2}{3\sqrt{3}} \frac{\alpha_D(3-2\alpha_D)(3-4\alpha_D)g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} \frac{m_{\Lambda^0} + m_K - m_N}{m_\Lambda^2 + 2m_K T_N - (m_N - m_K)^2} \\ &- \frac{1}{2\sqrt{3}g_A^2} \frac{(3-2\alpha_D)g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{1}{m_N} + \frac{1}{6g_A^2} \frac{(3-4\alpha_D)g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} \frac{1}{m_N} = \\ &= -1.93 \times 10^{-6} \text{ MeV}^{-3}, \\ B_{p\Lambda^0 K^+}^{pp} &= +\frac{1}{3\sqrt{3}} \frac{(3-2\alpha_D)^3 g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{m_{\Lambda^0} + m_K - m_N}{m_\Lambda^2 + 2m_K T_N - (m_N - m_K)^2} \\ &+ \frac{1}{\sqrt{3}} \frac{(2\alpha_D-1)^2(3-2\alpha_D)g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} \\ &- \frac{1}{\sqrt{3}g_A^2} \frac{(3-2\alpha_D)g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{1}{m_N} = \\ &= -0.35 \times 10^{-6} \text{ MeV}^{-3}, \end{aligned} \quad (2.3)$$

where $g_{\pi NN} = 13.21$ is the coupling constant of πNN interactions [31], $g_A = 1.267$ is the axial-vector coupling constant, renormalized by strong low-energy interactions, $F_\pi = 92.4 \text{ MeV}$ is the PCAC constant and $\alpha_D = D/(D+F) = 0.635$ is the Gell–Mann parameter [7], $T_N = E_N - m_N = (m_{\Lambda^0} + m_K - m_N)/2 = 335 \text{ MeV}$ is the kinetic energy of the relative motion of the protons at threshold in the center of mass frame, calculated for $m_N = 940 \text{ MeV}$, $m_{\Lambda^0} = 1116 \text{ MeV}$ and $m_K = 494 \text{ MeV}$. We have used also $m_\Sigma = 1193 \text{ MeV}$, $m_\pi = 140 \text{ MeV}$ and $m_\eta = 550 \text{ MeV}$ [7].

By a Fierz transformation we get

$$\begin{aligned}
[\bar{\Lambda}^0(x)p(x)][\bar{p}(x)i\gamma^5 p(x)] &= -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 p^c(x)][\bar{p}^c(x)p(x)] \\
&\quad + \frac{1}{4} i [\bar{\Lambda}^0(x)\gamma_\mu p^c(x)][\bar{p}^c(x)\gamma^\mu \gamma^5 p(x)] + \dots, \\
[\bar{\Lambda}^0(x)i\gamma^5 p(x)][\bar{p}(x)p(x)] &= -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 p^c(x)][\bar{p}^c(x)p(x)] \\
&\quad - \frac{1}{4} i [\bar{\Lambda}^0(x)\gamma_\mu p^c(x)][\bar{p}^c(x)\gamma^\mu \gamma^5 p(x)] + \dots \quad (2.4)
\end{aligned}$$

Hence, the effective Lagrangian of the transition $pp \rightarrow p\Lambda^0 K^+$ is equal to

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{pp \rightarrow p\Lambda^0 K^+}(x) &= -\frac{1}{4} C_{(p\Lambda^0)1S_0}^{(pp)3P_0} K^+ [\bar{\Lambda}^0(x)i\gamma^5 p^c(x)][\bar{p}^c(x)p(x)] K^{+\dagger}(x) \\
&\quad + \frac{1}{4} C_{(p\Lambda^0)3S_1}^{(pp)3P_1} K^+ [\bar{\Lambda}^0(x)i\gamma^\mu p^c(x)][\bar{p}^c(x)\gamma_\mu \gamma^5 p(x)] K^{+\dagger}(x), \quad (2.5)
\end{aligned}$$

where the first and the second terms describe the production of the $p\Lambda^0$ pair in the 1S_0 and 3S_1 state by the pp pair in the 3P_0 and 3P_1 state, respectively. The effective coupling constants $C_{(p\Lambda^0)1S_0}^{(pp)3P_0} K^+$ and $C_{(p\Lambda^0)3S_1}^{(pp)3P_1} K^+$ are equal to

$$\begin{aligned}
C_{(p\Lambda^0)1S_0}^{(pp)3P_0} K^+ &= A_{p\Lambda^0 K^+}^{pp} + B_{p\Lambda^0 K^+}^{pp} = -2.28 \times 10^{-6} \text{ MeV}^{-3}, \\
C_{(p\Lambda^0)3S_1}^{(pp)3P_1} K^+ &= A_{p\Lambda^0 K^+}^{pp} - B_{p\Lambda^0 K^+}^{pp} = -1.58 \times 10^{-6} \text{ MeV}^{-3} \quad (2.6)
\end{aligned}$$

The total cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ is given by

$$\begin{aligned}
\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon) &= \sigma^{(pp)3P_0 \rightarrow (p\Lambda^0)1S_0 K^+}(\varepsilon) + \sigma^{(pp)3P_1 \rightarrow (p\Lambda^0)3S_1 K^+}(\varepsilon) = \\
&= \frac{1}{128\pi^2} \frac{\varepsilon^2}{m_K} \frac{p_0}{E_0} \left(\frac{m_K m_{\Lambda^0} m_N}{m_K + m_{\Lambda^0} + m_N} \right)^{3/2} \\
&\times \left(\frac{1}{3} \left| C_{(p\Lambda^0)1S_0}^{(pp)3P_0} K^+ \right|^2 \left| f_{(pp)3P_0}(p_0) \right|^2 + \frac{2}{3} \left| C_{(p\Lambda^0)3S_1}^{(pp)3P_1} K^+ \right|^2 \left| f_{(pp)3P_1}(p_0) \right|^2 \right) \Omega_{p\Lambda^0 K^+}(\varepsilon), \quad (2.7)
\end{aligned}$$

where $\varepsilon = 2E_N - m_K - m_{\Lambda^0} - m_N$ is the excess energy for the relative energy of the pp pair in the center of mass frame $E_N = \sqrt{p^2 + m_N^2}$. It is measured in MeV. The function $\Omega_{p\Lambda^0 K^+}(\varepsilon)$ is related to the account for the final-state interaction in the $p\Lambda^0$ and pK^+ channels defined in Appendix A (see also [10]). For $\varepsilon \geq 6.68$ MeV the contribution of the Coulomb repulsion in the K^+p pair makes up about 15 %. Therefore for $\varepsilon \geq 6.68$ MeV we carry out the calculation of $\Omega_{p\Lambda^0 K^+}(\varepsilon)$ at the neglect of the Coulomb repulsion.

The amplitudes $f_{(pp)3P_0}(p_0)$ and $f_{(pp)3P_1}(p_0)$ describe the pp rescattering in the 3P_0 and 3P_1 state at threshold of the final $p\Lambda^0 K^+$ state, respectively. The detailed procedure for the calculation of the amplitudes $f_{(pp)3P_0}(p_0)$ and $f_{(pp)3P_1}(p_0)$ is expounded in [19]. Following this procedure one gets

$$\begin{aligned}
|f_{(pp)3P_0}(p_0)| &= \left| \left\{ 1 + \frac{C_{(pp)3P_0}(p_0)}{8\pi^2} \frac{p_0^3}{E_N} \left[\ln\left(\frac{E_N + p_0}{E_N - p_0}\right) + i\pi \right] \right\}^{-1} \right| = 0.15, \\
|f_{(pp)3P_1}(p_0)| &= \left| \left\{ 1 - \frac{C_{(pp)3P_1}(p_0)}{12\pi^2} \frac{p_0^3}{E_N} \left[\ln\left(\frac{E_N + p_0}{E_N - p_0}\right) - i\pi \right] \right\}^{-1} \right| = 0.24, \quad (2.8)
\end{aligned}$$

where the effective coupling constants $C_{(pp)_{3P_0}}(p_0)$ and $C_{(pp)_{3P_1}}(p_0)$ are defined by the π^0 – and η –meson exchanges¹.

For the relative momentum of the pp pair $p_0 = \sqrt{T_N(T_N + 2m_N)} = 861 \text{ MeV}$ at threshold of the reaction $pp \rightarrow p\Lambda^0 K^+$, the effective coupling constants $C_{(pp)_{3P_0}}(p_0)$ and $C_{(pp)_{3P_1}}(p_0)$ amount to [19]

$$\begin{aligned} C_{(pp)_{3P_1}}(p_0) = C_{(pp)_{3P_0}}(p_0) &= \frac{g_{\pi NN}^2}{4p_0^2} \ln\left(1 + \frac{4p_0^2}{m_\pi^2}\right) + (3 - 4\alpha_D)^2 \frac{g_{\pi NN}^2}{12p_0^2} \ln\left(1 + \frac{4p_0^2}{m_\eta^2}\right) = \\ &= 3.05 \times 10^{-4} \text{ MeV}^{-2}, \end{aligned} \quad (2.9)$$

The cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ is equal to

$$\begin{aligned} \sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon) &= \sigma^{(pp)_{3P_0} \rightarrow (p\Lambda^0)_{1S_0} K^+}(\varepsilon) + \sigma^{(pp)_{3P_1} \rightarrow (p\Lambda^0)_{3S_1} K^+}(\varepsilon) = \\ &= (1.50 \varepsilon^2 + 3.70 \varepsilon^2) \Omega_{p\Lambda^0 K^+}(\varepsilon) \text{ nb} = 3.49 \varepsilon^2 \text{ nb}, \end{aligned} \quad (2.10)$$

where $\Omega_{p\Lambda^0 K^+}(\varepsilon) = 0.67$ is calculated at $\varepsilon = 6.68 \text{ MeV}$ (see Appendix A). The theoretical value (2.10) agrees with the experimental data $\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon)_{\text{exp}} = 3.68 \varepsilon^2 \text{ nb}$ [20] within an accuracy better than 6 %.

Now we take into account the contribution of the ρ^0 – and ω^0 –mesons. The effective Lagrangian of the $pp \rightarrow p\Lambda^0 K^+$ transition, caused by the vector–meson exchanges, is equal to²

$$\begin{aligned} \delta \mathcal{L}_{\text{eff}}^{pp \rightarrow p\Lambda^0 K^+}(x) &= -\frac{1}{2\sqrt{3}} \frac{(3 - 2\alpha_D) g_{\pi NN} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\ &\quad \times i [\bar{\Lambda}^0(x) \gamma_\mu \gamma^5 p(x)] [\bar{p}(x) \gamma^\mu p(x)] K^{+\dagger}(x). \end{aligned} \quad (2.11)$$

By a Fierz transformation we reduce the effective interaction to the form

$$\begin{aligned} i [\bar{\Lambda}^0(x) \gamma_\mu \gamma^5 p(x)] [\bar{p}(x) \gamma^\mu p(x)] &\rightarrow - [\bar{\Lambda}^0(x) i \gamma^5 p^c(x)] [\bar{p}^c(x) p(x)] \\ &\quad - \frac{1}{2} [\bar{\Lambda}^0(x) i \gamma_\mu p^c(x)] [\bar{p}^c(x) \gamma^\mu \gamma^5 p(x)] + \dots \end{aligned} \quad (2.12)$$

The contributions of the ρ^0 – and ω^0 –meson exchanges to the coefficients $C_{(p\Lambda^0)_{1S_0} K^+}^{(pp)_{3P_0}}$ and $C_{(p\Lambda^0)_{3S_1} K^+}^{(pp)_{3P_1}}$ are equal to

$$\begin{aligned} \delta C_{(p\Lambda^0)_{1S_0} K^+}^{(pp)_{3P_0}} &= -\frac{2}{\sqrt{3}} \frac{(3 - 2\alpha_D) g_{\pi NN} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} = -0.30 \times 10^{-6} \text{ MeV}^{-3}, \\ \delta C_{(p\Lambda^0)_{3S_1} K^+}^{(pp)_{3P_1}} &= +\frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_D) g_{\pi NN} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} = +0.15 \times 10^{-6} \text{ MeV}^{-3}. \end{aligned} \quad (2.13)$$

¹By using our expression for the amplitude $f_{(pp)_{3P_0}}(p_0)$ and the results obtained in [16] we can estimate the values of the phase shift $\delta_{3P_0}(p_0)$ and the inelasticity $\eta_{3P_0}(p_0)$ of pp scattering in the 3P_0 state. We get $\delta_{3P_0}(p_0) = -63.0^\circ$ and $\eta_{3P_0}(p_0) = 0.80$. This does not contradict the SAID analysis of the experimental data [32, 33]: $\delta_{3P_0}(p_0) = -62.7^\circ$ and $\eta_{3P_0}(p_0) = 0.65$.

²Since we compare are results with those by Fäldt and Wilkin [6], for the calculation of the contribution of vector–meson exchanges we follow Fäldt and Wilkin [6] and treat vector mesons as Yang–Mills fields [11, 14]. The interactions of vector mesons with hadronic fields are defined by a minimal extension [11, 14]. For the derivation of the Lagrangian of the interactions of baryons with vector mesons one can use the Lagrangian (D.4) of Ref.[19] with the replacements $g_{\pi NN} \rightarrow g_\rho/2$, $\alpha_D \rightarrow 0$, $i\gamma^5 \rightarrow \gamma^\mu$, $\pi(x) \rightarrow \rho_\mu(x)$ and $\eta(x) \rightarrow \omega_\mu(x)/\sqrt{3}$.

For the numerical calculation we set $m_\rho = m_\omega = 780 \text{ MeV}$ [7]. The contributions of the ρ^0 - and ω^0 -meson exchanges to the effective coupling constants $C_{(pp)_{3P_0}}(p_0)$ and $C_{(pp)_{3P_1}}(p_0)$ amount to

$$\begin{aligned}\delta C_{(pp)_{3P_0}} &= -\frac{g_\rho^2}{2p_0^2} \ell n \left(1 + \frac{4p_0^2}{m_\rho^2}\right) = -0.43 \times 10^{-4} \text{ MeV}^{-2}, \\ \delta C_{(pp)_{3P_1}} &= +\frac{g_\rho^2}{4p_0^2} \ell n \left(1 + \frac{4p_0^2}{m_\rho^2}\right) = +0.22 \times 10^{-4} \text{ MeV}^{-2}.\end{aligned}\quad (2.14)$$

This gives $|f_{(pp)_{3P_0}}(p_0)| = 0.16$ and $|f_{(pp)_{3P_1}}(p_0)| = 0.22$. The cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ with the ρ^0 - and ω^0 -meson exchanges is

$$\begin{aligned}\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon) &= \sigma^{(pp)_{3P_0} \rightarrow (p\Lambda^0)_{1S_0} K^+}(\varepsilon) + \sigma^{(pp)_{3P_1} \rightarrow (p\Lambda^0)_{3S_1} K^+}(\varepsilon) = \\ &= (2.19 \varepsilon^2 + 2.55 \varepsilon^2) \Omega_{p\Lambda^0 K^+}(\varepsilon) \text{ nb} = 3.18 \varepsilon^2 \text{ nb}.\end{aligned}\quad (2.15)$$

Thus, the ρ^0 - and ω^0 -meson exchanges decrease the value of the cross section for the reaction $pp \rightarrow p\Lambda^0 K^+$ by about 9% and lead to the agreement with the experimental data $\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon)_{\text{exp}} = 3.68 \varepsilon^2 \text{ nb}$ [20] with an accuracy about 16%.

3 Cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$ near threshold

The pn pair in the reaction $pn \rightarrow n\Lambda^0 K^+$ can interact both in the isospin-triplet $(pn)_{I=1}$ state and isospin-singlet $(pn)_{I=0}$ state. According to the *generalised Pauli exclusion principle*, the pn pair with isospin $I = 1$ can be in the 3P_0 and 3P_1 state with the $n\Lambda^0$ pair in the 1S_0 and 3S_1 state, respectively, whereas the pn pair in the state with isospin $I = 0$ can be in the 1P_1 state only with the $n\Lambda^0$ pair in the 3S_1 state.

A complete set of Feynman diagrams for the $pn \rightarrow n\Lambda^0 K^+$ transition near threshold of the final state is depicted in Fig.3 and Fig.4. The wave functions of the initial and final states we take in the form

$$\begin{aligned}|p(\vec{p}_1, \sigma_1) n(\vec{p}_2, \sigma_2)\rangle &= a_p^\dagger(\vec{p}_1, \sigma_1) a_n^\dagger(\vec{p}_2, \sigma_2) |0\rangle, \\ |n(\vec{k}_n, \sigma_n) \Lambda^0(\vec{k}_\Lambda, \sigma_\Lambda) K^+(\vec{Q}_K)\rangle &= a_n^\dagger(\vec{k}_n, \sigma_n) a_\Lambda^\dagger(\vec{k}_\Lambda, \sigma_\Lambda) c_{K^+}^\dagger(\vec{Q}_K) |0\rangle,\end{aligned}\quad (3.1)$$

The matrix element of the $pn \rightarrow n\Lambda^0 K^+$ transition, defined by the Feynman diagrams in Fig.3 and Fig.4, is described by the effective local Lagrangian of the transition $pn \rightarrow n\Lambda^0 K^+$:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{pn \rightarrow n\Lambda^0 K^+}(x) &= (A_{n\Lambda^0 K^+}^{pn} [\bar{\Lambda}^0(x) n(x)] [\bar{n}(x) i\gamma^5 p(x)] + B_{n\Lambda^0 K^+}^{pn} [\bar{\Lambda}^0(x) p(x)] [\bar{n}(x) i\gamma^5 n(x)] \\ &+ C_{n\Lambda^0 K^+}^{pn} [\bar{\Lambda}^0(x) i\gamma^5 n(x)] [\bar{n}(x) p(x)] + D_{n\Lambda^0 K^+}^{pn} [\bar{\Lambda}^0(x) i\gamma^5 p(x)] [\bar{n}(x) n(x)]), \\ &\times K^{+\dagger}(x),\end{aligned}\quad (3.2)$$

where the effective coupling constants $A_{n\Lambda^0 K^+}^{pn}$, $B_{n\Lambda^0 K^+}^{pn}$, $C_{n\Lambda^0 K^+}^{pn}$ and $D_{n\Lambda^0 K^+}^{pn}$ are equal to

$$A_{n\Lambda^0 K^+}^{pn} = -\frac{2}{\sqrt{3}} \frac{(3 - 2\alpha_D) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K}$$

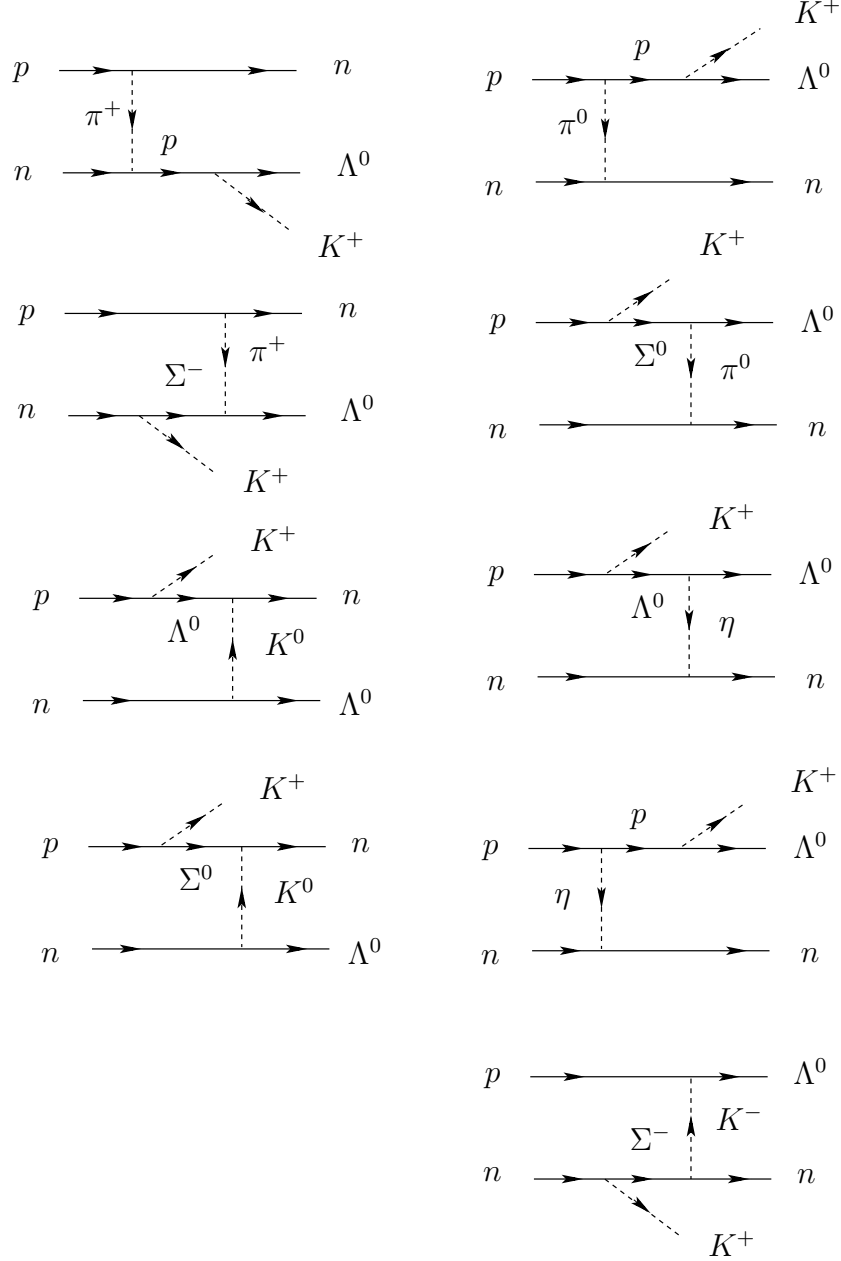


Figure 3: The set of Feynman diagrams contributing to the effective coupling constants of the $pn \rightarrow n\Lambda^0 K^+$ transition in the one-pseudoscalar meson exchange approximation.

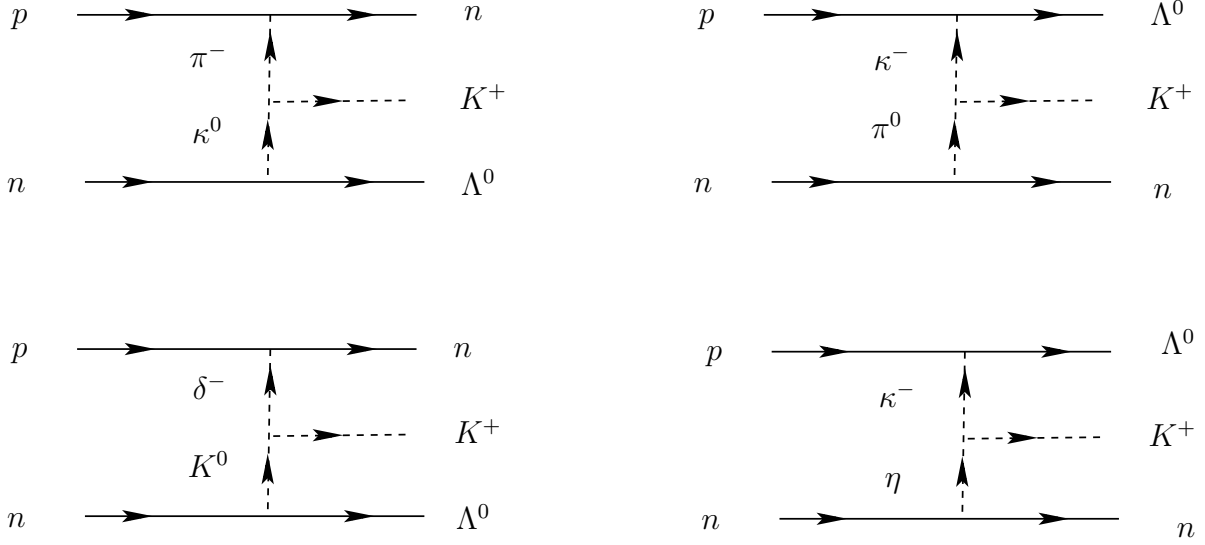


Figure 4: The set of Feynman diagrams contributing to the effective coupling constants of the $pn \rightarrow n\Lambda^0 K^+$ transition in the one-scalar meson exchange approximation.

$$\begin{aligned}
& + \frac{4}{\sqrt{3}} \frac{\alpha_D (2\alpha_D - 1) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} \\
& + \frac{1}{\sqrt{3} g_A^2} \frac{1}{m_N} \frac{(3 - 2\alpha_D) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} = 0.23 \times 10^{-6} \text{ MeV}^{-3}, \\
B_{n\Lambda^0 K^+}^{pn} & = + \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_D) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\
& - \frac{2}{\sqrt{3}} \frac{\alpha_D (2\alpha_D - 1) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} \\
& + \frac{2}{3\sqrt{3}} \frac{\alpha_D (3 - 2\alpha_D)(3 - 4\alpha_D) g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} \frac{m_{\Lambda^0} + m_K - m_N}{m_\Lambda^2 + 2m_K T_N - (m_N - m_K)^2} \\
& - \frac{1}{3\sqrt{3}} \frac{(3 - 2\alpha_D)(3 - 4\alpha_D)^2 g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\
& + \frac{1}{2\sqrt{3} g_A^2} \frac{1}{m_N} \frac{(3 - 2\alpha_D) g_{\pi NN}^3}{m_\pi^2 + 2m_N T_N} + \frac{1}{6\sqrt{3} g_A^2} \frac{1}{m_N} \frac{(3 - 2\alpha_D)(3 - 4\alpha_D) g_{\pi NN}^3}{m_\eta^2 + 2m_N T_N} = \\
& = 2.52 \times 10^{-6} \text{ MeV}^{-3}, \\
C_{n\Lambda^0 K^+}^{pn} & = - \frac{1}{3\sqrt{3}} \frac{(3 - 2\alpha_D)^3 g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{m_{\Lambda^0} + m_K - m_N}{m_\Lambda^2 + 2m_K T_N - (m_N - m_K)^2} \\
& + \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_D)(2\alpha_D - 1)^2 g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} \\
& + \frac{1}{\sqrt{3} g_A^2} \frac{1}{m_N} \frac{(3 - 2\alpha_D) g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} = 0.51 \times 10^{-6} \text{ MeV}^{-3}, \\
D_{n\Lambda^0 K^+}^{pn} & = \frac{2}{\sqrt{3}} \frac{(3 - 2\alpha_D)(2\alpha_D - 1)^2 g_{\pi NN}^3}{m_K^2 + 2m_{\Lambda^0} T_N - (m_{\Lambda^0} - m_N)^2} \frac{m_\Sigma + m_K - m_N}{m_\Sigma^2 + 2m_K T_N - (m_N - m_K)^2} = \\
& = 0.17 \times 10^{-6} \text{ MeV}^{-3}. \tag{3.3}
\end{aligned}$$

Since at low energies the $n\Lambda^0$ pair can be produced in the 1S_0 and 3S_1 state only, we have to extract these interactions from the effective Lagrangian Eq.(3.2) by a Fierz transformation. We get

$$\begin{aligned}
& [\bar{\Lambda}^0(x)n(x)][\bar{n}(x)i\gamma^5 p(x)] = -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 n^c(x)][\bar{n}^c(x)p(x)] \\
& + \frac{1}{4} [\bar{\Lambda}^0(x)i\gamma_\mu n^c(x)][\bar{n}^c(x)\gamma^\mu \gamma^5 p(x)] - \frac{1}{8} [\bar{\Lambda}^0(x)i\sigma_{\mu\nu} n^c(x)][\bar{n}^c(x)\sigma^{\mu\nu} \gamma^5 p(x)] + \dots, \\
& [\bar{\Lambda}^0(x)p(x)][\bar{n}(x)i\gamma^5 n(x)] = -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 n^c(x)][\bar{n}^c(x)p(x)] \\
& + \frac{1}{4} [\bar{\Lambda}^0(x)i\gamma_\mu n^c(x)][\bar{n}^c(x)\gamma^\mu \gamma^5 p(x)] + \frac{1}{8} [\bar{\Lambda}^0(x)i\sigma_{\mu\nu} n^c(x)][\bar{n}^c(x)\sigma^{\mu\nu} \gamma^5 p(x)] + \dots, \\
& [\bar{\Lambda}^0(x)i\gamma^5 n(x)][\bar{n}(x)p(x)] = -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 n^c(x)][\bar{n}^c(x)p(x)] \\
& - \frac{1}{4} [\bar{\Lambda}^0(x)i\gamma_\mu n^c(x)][\bar{n}^c(x)\gamma^\mu \gamma^5 p(x)] - \frac{1}{8} [\bar{\Lambda}^0(x)i\sigma_{\mu\nu} n^c(x)][\bar{n}^c(x)\sigma^{\mu\nu} \gamma^5 p(x)] + \dots, \\
& [\bar{\Lambda}^0(x)i\gamma^5 p(x)][\bar{n}(x)n(x)] = -\frac{1}{4} [\bar{\Lambda}^0(x)i\gamma^5 n^c(x)][\bar{n}^c(x)p(x)] \\
& - \frac{1}{4} [\bar{\Lambda}^0(x)i\gamma_\mu n^c(x)][\bar{n}^c(x)\gamma^\mu \gamma^5 p(x)] + \frac{1}{8} [\bar{\Lambda}^0(x)i\sigma_{\mu\nu} n^c(x)][\bar{n}^c(x)\sigma^{\mu\nu} \gamma^5 p(x)] + \dots \quad (3.4)
\end{aligned}$$

Hence, the effective Lagrangian of the $pn \rightarrow n\Lambda^0 K^+$ transition is equal to

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{pn \rightarrow n\Lambda^0 K^+}(x) = & -\frac{1}{2} C_{(n\Lambda^0)1S_0}^{(pn)3P_0} K^+ [\bar{\Lambda}^0(x)i\gamma^5 n^c(x)][\bar{n}^c(x)p(x)] K^{+\dagger}(x) \\
& + \frac{1}{2} C_{(n\Lambda^0)3S_1}^{(pn)3P_1} K^+ [\bar{\Lambda}^0(x)i\gamma_\mu n^c(x)][\bar{n}^c(x)\gamma^\mu \gamma^5 p(x)] K^{+\dagger}(x) \\
& - \frac{1}{4} C_{(n\Lambda^0)3S_1}^{(pn)1P_1} K^+ [\bar{\Lambda}^0(x)i\sigma_{\mu\nu} n^c(x)][\bar{n}^c(x)\sigma^{\mu\nu} \gamma^5 p(x)] K^{+\dagger}(x), \quad (3.5)
\end{aligned}$$

where the first and the second terms describe the production of the $n\Lambda^0$ pair in the 1S_0 and 3S_1 state by the pn pair in the isospin-triplet and the 3P_0 and 3P_1 states, respectively, the third term corresponds to the interaction of the pn pair in the isospin-singlet and the 1P_1 state. At threshold for the pn pair in the 1P_1 state with isospin $I = 0$ the $n\Lambda^0$ pair produces itself in the 3S_1 state.

The effective coupling constants are equal to

$$\begin{aligned}
C_{(n\Lambda^0)1S_0}^{(pn)3P_0} K^+ &= \frac{1}{2} (A_{n\Lambda^0 K^+}^{pn} + B_{n\Lambda^0 K^+}^{pn} + C_{n\Lambda^0 K^+}^{pn} + D_{n\Lambda^0 K^+}^{pn}) = \\
&= +1.72 \times 10^{-6} \text{ MeV}^{-3}, \\
C_{(n\Lambda^0)3S_1}^{(pn)3P_1} K^+ &= \frac{1}{2} (A_{n\Lambda^0 K^+}^{pn} + B_{n\Lambda^0 K^+}^{pn} - C_{n\Lambda^0 K^+}^{pn} - D_{n\Lambda^0 K^+}^{pn}) = \\
&= +1.04 \times 10^{-6} \text{ MeV}^{-3}, \\
C_{(n\Lambda^0)3S_1}^{(pn)1P_1} K^+ &= \frac{1}{2} (A_{n\Lambda^0 K^+}^{pn} - B_{n\Lambda^0 K^+}^{pn} + C_{n\Lambda^0 K^+}^{pn} - D_{n\Lambda^0 K^+}^{pn}) = \\
&= -1.95 \times 10^{-6} \text{ MeV}^{-3}, \quad (3.6)
\end{aligned}$$

The total cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$ is given by

$$\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) = \frac{1}{2} (\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) + \sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon)), \quad (3.7)$$

where the cross sections $\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon)$ and $\sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon)$ are defined by

$$\begin{aligned}\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) &= 2(\sigma^{(pn)_{3P_0} \rightarrow (n\Lambda^0)_{1S_0} K^+}(\varepsilon) + \sigma^{(pn)_{3P_1} \rightarrow (n\Lambda^0)_{3S_1} K^+}(\varepsilon)) = \\ &= \frac{1}{64\pi^2} \frac{\varepsilon^2}{m_K} \frac{p_0}{E_0} \left(\frac{m_K m_{\Lambda^0} m_N}{m_K + m_{\Lambda^0} + m_N} \right)^{3/2} \\ &\times \left(\frac{1}{3} \left| C_{(n\Lambda^0)_{1S_0} K^+}^{(pn)_{3P_0}} \right|^2 \left| f_{(pn)_{3P_0}}(p_0) \right|^2 + \frac{2}{3} \left| C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{3P_1}} \right|^2 \left| f_{(pn)_{3P_1}}(p_0) \right|^2 \right) \Omega_{n\Lambda^0 K^+}(\varepsilon), \\ \sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon) &= 2\sigma^{(pn)_{1P_1} \rightarrow (n\Lambda^0)_{1S_0} K^+}(\varepsilon) = \\ &= \frac{1}{64\pi^2} \frac{\varepsilon^2}{m_K} \frac{p_0}{E_0} \left(\frac{m_K m_{\Lambda^0} m_N}{m_K + m_{\Lambda^0} + m_N} \right)^{3/2} \frac{1}{3} \left| C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{1P_1}} \right|^2 \left| f_{(pn)_{1P_1}}(p_0) \right|^2 \Omega_{n\Lambda^0 K^+}(\varepsilon). \quad (3.8)\end{aligned}$$

The function $\Omega_{n\Lambda^0 K^+}(\varepsilon)$ takes into account the final-state interaction in the $n\Lambda^0$ channel (see Appendix A). The amplitude $f_{(pn)_{1P_1}}(p_0)$ describes the rescattering of the pn pair in the isospin-singlet and 1P_1 state.

Since the amplitudes of the pn rescattering in the 3P_0 and 3P_1 state, $f_{(pn)_{3P_0}}(p_0)$ and $f_{(pn)_{3P_1}}(p_0)$, are equal to the amplitudes of the pp rescattering, the value of the cross section for the reaction $(pn)_{I=1} \rightarrow n\Lambda^0 K^+$ is equal to

$$\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) = (1.71 \varepsilon^2 + 3.21 \varepsilon^2) \Omega_{n\Lambda^0 K^+}(\varepsilon) \text{ nb} = 3.30 \varepsilon^2 \text{ nb}, \quad (3.9)$$

where $\Omega_{n\Lambda^0 K^+}(\varepsilon) = 0.67$ for $\varepsilon = 6.68 \text{ MeV}$ (see Appendix A).

For the cross sections of the reactions $(pn)_{I=1} \rightarrow n\Lambda^0 K^+$ and $pp \rightarrow p\Lambda^0 K^+$, defined by Eqs.(2.10) and (3.9), we get the ratio

$$R^{(I=1)} = \frac{\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon)}{\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon)} \simeq 1, \quad (3.10)$$

agreeing well with isospin-invariance of strong pN interactions.

According to [19], the result of the calculation of the amplitude $f_{(pn)_{1P_1}}(p_0)$ is

$$|f_{(pn)_{1P_1}}(p_0)| = \left| \left\{ 1 - \frac{C_{(pn)_{1P_1}}(p_0)}{24\pi^2} \frac{p_0^3}{E_N} \left[\ell n \left(\frac{E_N + p_0}{E_N - p_0} \right) - i\pi \right] \right\}^{-1} \right| = 0.49, \quad (3.11)$$

where the coupling constant $C_{(pn)_{1P_1}}(p_0)$ is defined by the π - and η -meson exchanges and is equal to $C_{(pn)_{1P_1}}(p_0) = C_{(pn)_{3P_1}}(p_0) = 3.05 \times 10^{-4} \text{ MeV}^{-2}$ [19]. The cross section for the reaction $(pn)_{I=0} \rightarrow n\Lambda^0 K^+$ amounts to

$$\sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon) = 23.50 \varepsilon^2 \Omega_{n\Lambda^0 K^+}(\varepsilon) \text{ nb} = 15.75 \varepsilon^2 \text{ nb}. \quad (3.12)$$

For the total cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$ we get

$$\begin{aligned}\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) &= \frac{1}{2}(\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) + \sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon)) = \\ &= \Delta\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) + \sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) = 9.52 \varepsilon^2 \text{ nb}, \quad (3.13)\end{aligned}$$

where we have denoted

$$\Delta\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) = \frac{1}{2}(\sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon) - \sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon)) = 6.22 \varepsilon^2 \text{ nb}. \quad (3.14)$$

Now we can take into account the contribution of the vector meson exchanges. Unlike the reaction $pp \rightarrow p\Lambda^0 K^+$ the contribution of the ρ^0 – and ω^0 –meson exchanges is cancelled and there are only the contributions of the charged ρ –meson exchange. The effective Lagrangian, caused by the charged ρ –meson exchanges, is equal to

$$\begin{aligned} \delta\mathcal{L}_{\text{eff}}^{pn \rightarrow n\Lambda^0 K^+}(x) &= -\frac{1}{2\sqrt{3}} \frac{(3-2\alpha_D) g_{\pi N5} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} \\ &\times [\bar{\Lambda}^0(x) i \gamma_\mu \gamma^5 n(x)] [\bar{n}(x) \gamma^\mu p(x)] K^{+\dagger}(x). \end{aligned} \quad (3.15)$$

By a Fierz transformation

$$\begin{aligned} [\bar{\Lambda}^0(x) i \gamma_\mu \gamma^5 n(x)] [\bar{n}(x) \gamma^\mu p(x)] &= -\frac{1}{4} [\bar{\Lambda}^0(x) i \gamma^5 n^c(x)] [\bar{n}^c(x) p(x)] \\ &- \frac{1}{4} [\bar{\Lambda}^0(x) i \gamma_\mu n^c(x)] [\bar{n}^c(x) \gamma^\mu \gamma^5 p(x)] + \dots \end{aligned} \quad (3.16)$$

we obtain the contributions of the charged ρ –meson exchanges to the effective coupling constants $C_{(n\Lambda^0)_{1S_0} K^+}^{(pn)_{3P_0}}$, $C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{3P_1}}$ and $C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{1P_1}}$

$$\begin{aligned} \delta C_{(n\Lambda^0)_{1S_0} K^+}^{(pn)_{3P_0}} &= -\frac{1}{4\sqrt{3}} \frac{(3-2\alpha_D) g_{\pi NN} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} = -0.04 \times 10^{-6} \text{ MeV}^{-3}, \\ \delta C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{3P_1}} &= +\frac{1}{4\sqrt{3}} \frac{(3-2\alpha_D) g_{\pi NN} g_\rho^2}{m_\rho^2 + 2m_N T_N} \frac{1}{m_N + m_{\Lambda^0} + m_K} = +0.04 \times 10^{-6} \text{ MeV}^{-3}, \\ \delta C_{(n\Lambda^0)_{3S_1} K^+}^{(pn)_{1P_1}} &= 0. \end{aligned} \quad (3.17)$$

The contributions of the charged ρ –meson exchanges to the effective coupling constants $C_{(pn)_{3P_0}}(p_0)$ and $C_{(pn)_{3P_1}}(p_0)$ amount to

$$\begin{aligned} \delta C_{(pn)_{3P_0}}(p_0) &= -\frac{g_\rho^2}{2p_0^2} \ell n \left(1 + \frac{4p_0^2}{m_\rho^2} \right) = -0.43 \times 10^{-4} \text{ MeV}^{-2}, \\ \delta C_{(pn)_{3P_1}}(p_0) &= +\frac{g_\rho^2}{4p_0^2} \ell n \left(1 + \frac{4p_0^2}{m_\rho^2} \right) = +0.22 \times 10^{-4} \text{ MeV}^{-2}. \end{aligned} \quad (3.18)$$

This gives $|f_{(pn)_{3P_0}}(p_0)| = 0.16$ and $|f_{(pn)_{3P_1}}(p_0)| = 0.22$. The cross section for the reaction $(pn)_{I=1} \rightarrow n\Lambda^0 K^+$ with the contribution of the vector–meson exchanges is

$$\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) = (1.86 \varepsilon^2 + 2.91 \varepsilon^2) \Omega_{n\Lambda^0 K^+}(\varepsilon) \text{ nb} = 3.20 \varepsilon^2 \text{ nb}. \quad (3.19)$$

This gives the following value of the ratio $R^{(I=1)}$:

$$R^{(I=1)} = \frac{\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon)}{\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon)} \simeq 1, \quad (3.20)$$

which is in agreement with isospin invariance of strong pN interactions.

The vector–meson exchanges lead to the decrease of the cross section for the reaction $(pn)_{I=1} \rightarrow n\Lambda^0 K^+$ by about 3%. Thus, the total cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$ is equal to

$$\begin{aligned}\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) &= \frac{1}{2}(\sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) + \sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon)) = \\ &= \Delta\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) + \sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon) = \\ &= (6.28\varepsilon^2 + 3.20\varepsilon^2)\text{ nb} = 9.48\varepsilon^2\text{ nb}.\end{aligned}\tag{3.21}$$

Hence, due to the dominant contribution of the pn interaction in the isospin–singlet state, the vector–meson exchanges have practically no influence on the cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$. The decrease of the total cross section for the reaction $pn \rightarrow n\Lambda^0 K^+$, caused by the vector–meson exchanges, is about 0.4%.

4 Ratio of the cross sections

Using the results obtained above we can calculate the ratio of the cross sections for the reactions $pn \rightarrow n\Lambda^0 K^+$ and $pp \rightarrow p\Lambda^0 K^+$. For the theoretical cross sections, calculated with the account for vector–meson exchanges Eq.(3.12) and Eq.(2.15) and without vector–meson exchanges Eq.(3.12) and Eq.(2.10), we get

$$R = \frac{\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon)}{\sigma^{pp \rightarrow p\Lambda^0 K^+}(\varepsilon)} \simeq 3.\tag{4.1}$$

The obtained result confirms an enhancement of the pn interaction with respect to the pp interaction near observed by the ANKE–Collaboration at COSY [1]. In our analysis such an enhancement is due to the contribution of the pn interaction in the isospin–singlet state $\Delta\sigma^{pn \rightarrow n\Lambda^0 K^+}(\varepsilon) = \frac{1}{2}(\sigma^{(pn)_{I=0} \rightarrow n\Lambda^0 K^+}(\varepsilon) - \sigma^{(pn)_{I=1} \rightarrow n\Lambda^0 K^+}(\varepsilon)) \approx 6.3\text{ nb}$.

5 On the $N(1535)$ and $N(1650)$ resonance exchanges

As has been pointed out by Wilkin [8], an important contribution to the cross sections for the reactions $pN \rightarrow N\Lambda^0 K^+$ should come from the exchange with the $N(1535)$ resonance [34]. This is the $S_{11}(1535)$ resonance with the quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ [34].

The inclusion of the $N(1535)$ –resonance exchange demands the knowledge of its interactions with the octets of the ground baryons and the pseudoscalar mesons. Since the $N(1535)$ resonance is the member of octet [35], the effective Lagrangian of its interactions with the ground state baryons and the pseudoscalar mesons reads

$$\begin{aligned}\mathcal{L}_{\text{PBN}_1}(x) &= \sqrt{2}g_{\pi NN_1}\bar{p}(x)n_1(x)\pi^+(x) + \sqrt{2}g_{\pi NN_1}\bar{n}(x)p_1(x)\pi^-(x) \\ &+ g_{\pi NN_1}[\bar{p}(x)p_1(x) - \bar{n}(x)n_1(x)]\pi^0(x) \\ &+ \frac{1}{\sqrt{3}}(3 - 4\alpha_{D1})g_{\pi NN_1}[\bar{p}(x)p_1(x) + \bar{n}(x)n_1(x)]\eta(x) \\ &+ \frac{1}{\sqrt{3}}(3 - 2\alpha_{D1})g_{\pi NN_1}\bar{\Lambda}^0(x)p_1(x)K^-(x) + \dots + \text{h.c.},\end{aligned}\tag{5.1}$$

where $N_1(x) = (p_1(x), n_1(x))$ are the local interpolating field of the resonance $N(1535)$, which we treat as an elementary particle [21]–[27] as well as the $\Delta(1232)$ resonance [22, 23].

Due to the ambiguity of the parameters of the $N(1535)$ resonance the numerical value of the contribution of this resonance is rather ambiguous [7]. Below we use the following parameters: the mass $m_{N(1535)} = 1520 \text{ MeV}$, the width $\Gamma_{N(1535)} = 100 \text{ MeV}$ and the branching ratios $B(N(1535) \rightarrow N\pi) = 40 \%$ and $B(N(1535) \rightarrow N\eta) = 35 \%$. We leave 25 % of the total width for other decay channels such as $N(1535) \rightarrow \Delta(1232)\pi$ and so on [7]. For such a choice of the parameters the coupling constants are equal to $g_{\pi NN_1} = 0.53$ and $\alpha_{D1} = -0.52$.

For the reaction $pp \rightarrow p\Lambda^0 K^+$ the $N(1535)$ resonance gives the contribution only to the effective coupling constant $A_{p\Lambda^0 K^+}^{pp}$. We get

$$(\delta A_{p\Lambda^0 K^+}^{pp})_{N(1535)} = \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_{D1})g_{\pi NN} g_{\pi NN_1}^2}{m_{N_1} - m_{\Lambda^0} - m_K} \times \left[\frac{1}{m_\pi^2 + 2m_N T_N} + \frac{1}{3} \frac{(3 - 4\alpha_{D1})(3 - 4\alpha_D)}{m_\eta^2 + 2m_N T_N} \right] = -0.22 \times 10^{-6} \text{ MeV}^{-3}. \quad (5.2)$$

We can also take into account the contribution of the $N(1650)$ resonance (the $S_{11}(1650)$ resonance) with the quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, which is the octet partner of the resonances $\Lambda(1800)$ and $\Sigma(1750)$ [35]. The coupling constants of the $N(1650)$ resonance with the ground state baryons and the pseudoscalar mesons are equal to $g_{\pi NN_2} = 0.62$ and $\alpha_{D2} = 1.17$. They are calculated for $m_{N(1650)} = 1640 \text{ MeV}$, $B(N(1650) \rightarrow N\pi) = 70 \%$ and $B(N(1650) \rightarrow \Lambda^0 K) = 10 \%$ and $\Gamma_{N(1650)} = 170 \text{ MeV}$ [7].

The $N(1650)$ resonance as well as the $N(1535)$ resonance gives the contribution to the effective coupling constant $A_{p\Lambda^0 K^+}^{pp}$. The result is

$$(\delta A_{p\Lambda^0 K^+}^{pp})_{N(1650)} = \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_{D2})g_{\pi NN} g_{\pi NN_2}^2}{m_{N_2} - m_{\Lambda^0} - m_K} \times \left[\frac{1}{m_\pi^2 + 2m_N T_N} + \frac{1}{3} \frac{(3 - 4\alpha_{D1})(3 - 4\alpha_D)}{m_\eta^2 + 2m_N T_N} \right] = 0.08 \times 10^{-6} \text{ MeV}^{-3}. \quad (5.3)$$

Hence, the total contribution of the resonances $N(1535)$ and $N(1650)$ is about 8 %.

The values of the parameters of the resonances $N(1535)$ and $N(1650)$ are rather ambiguous. Nevertheless, as we show in Section 7 our choice of the parameters of the resonances $N(1535)$ and $N(1650)$ fits well the experimental data on the amplitude and the cross section for the reaction $\pi^- p \rightarrow \Lambda^0 K^0$ near threshold of the $\Lambda^0 K^0$ state.

We would like to accentuate that a cancellation of contributions of the resonances $N(1535)$ and $N(1650)$ occurs only in the Lagrangian approach, where the resonances are treated as elementary particles [21]–[27]. We cannot claim that such a cancellation between the contributions of these resonances should be within the approaches developed by Wilkin *et al.* [6, 8] or by Sibirtsev *et al.* [4, 9], where the $N(1535)$ and $N(1650)$ resonances were described within the relativistic generalisation of the Breit–Wigner approach [7].

Thus, treating the the resonances $N(1535)$ and $N(1650)$ as elementary particles we have shown that the total contribution of the resonances $N(1535)$ and $N(1650)$ to the matrix elements of the $pN \rightarrow N\Lambda^0 K^+$ transitions near threshold of the final state is less important in comparison with the contribution of the ground state baryon octet coupled to the octets of the pseudoscalar and scalar mesons.

6 Cross section for the reaction $pp \rightarrow pp\pi^0$

In this Section we show that the same procedure, which we have used for the analysis of the strangeness production in pN collisions near threshold, can be applied well to the description of the reaction $pp \rightarrow pp\pi^0$.

Near threshold the reaction $pp \rightarrow pp\pi^0$ is defined by a pure ${}^3P_0 \rightarrow {}^1S_0$ transition [36]–[39]. The effective Lagrangian responsible for the $(pp)_{3P_0} \rightarrow (pp)_{1S_0}\pi^0$ transition, calculated in the analogy with the effective Lagrangians (2.2) and (3.5), is

$$\mathcal{L}^{(pp \rightarrow pp\pi^0)}(x) = \frac{1}{4} C_{pp\pi^0}^{pp}(p_0) [\bar{p}(x) p^c(x)] [\bar{p}^c(x) i\gamma^5 p(x)] \pi^0(x). \quad (6.1)$$

The effective coupling constant $C_{pp\pi^0}^{pp}$ is equal to

$$\begin{aligned} C_{pp\pi^0}^{pp}(p_0) &= \frac{g_{\pi NN}^3}{m_N} \left[\left(1 - \frac{1}{g_A^2}\right) \frac{1}{m_\pi^2 + p_0^2} + \frac{1}{3} \frac{(3 - 4\alpha_D)}{m_\eta^2 + p_0^2} \left(3 - 4\alpha_D - \frac{1}{g_A^2}\right) \right] = \\ &= 6.04 \times 10^{-6} \text{ MeV}^{-3}, \end{aligned} \quad (6.2)$$

where $p_0 = 362.2 \text{ MeV}$ is a relative threshold momentum of the pp pair in the initial state.

For the cross section for the reaction $pp \rightarrow pp\pi^0$ we obtain the following expression

$$\begin{aligned} \sigma^{pp \rightarrow pp\pi^0}(\varepsilon) &= \frac{1}{192\pi^2} \frac{p_0}{E_0} \frac{\varepsilon^2}{m_\pi} \left(\frac{m_p^2 m_\pi}{2m_N + m_\pi} \right)^{3/2} |C_{pp\pi^0}^{pp}(p_0)|^2 |f_{(pp)_{3P_0}}(p_0)|^2 \Omega_{pp\pi^0}(\varepsilon) = \\ &= 0.43 \varepsilon^2 \Omega_{pp\pi^0}(\varepsilon) \mu\text{b}, \end{aligned} \quad (6.3)$$

where $|f_{(pp)_{3P_0}}(p_0)| = 0.38$ Eq.(2.8). The excess energy ε is related to the kinetic energy of the proton in the laboratory frame as $\varepsilon = \frac{1}{2} T_N - m_\pi$. It is measured in MeV. The function $\Omega_{pp\pi^0}(\varepsilon)$ is defined by the phase volume of the $pp\pi^0$ state. It is given by

$$\Omega_{pp\pi^0}(\varepsilon) = \frac{16}{\pi} \int_0^1 d\xi \xi^2 \sqrt{1 - \xi^2} |\psi_{pp}(\sqrt{m_N \varepsilon} \xi)|^2. \quad (6.4)$$

The wave function $\psi_{pp}(k)$, describing the final-state interaction in the pp channel accounting for the Coulomb repulsion, is determined by [40] (see also [41]):

$$\psi_{pp}(k) = e^{i\delta_{pp}^e(k)} \frac{\sin \delta_{pp}^e(k)}{-a_{pp}^e k C_0(k)} = \frac{C_0(k)}{1 - \frac{1}{2} a_{pp}^e r_{pp}^e k^2 + \frac{a_{pp}^e}{r_C} h(2kr_C) + i a_{pp}^e k C_0^2(k)}, \quad (6.5)$$

where we have denoted [40]

$$\begin{aligned} C_0^2(k) &= \frac{\pi}{kr_C} \frac{1}{e^{\pi/kr_C} - 1}, \\ h(2kr_C) &= -\gamma + \ell n(2kr_C) + \sum_{n=1}^{\infty} \frac{1}{n(1 + 4n^2 k^2 r_C^2)}, \\ \text{ctg} \delta_{pp}^e(k) &= \frac{1}{C_0^2(k) k} \left[-\frac{1}{a_{pp}^e} + \frac{1}{2} r_{pp}^e k^2 - \frac{1}{r_C} h(2kr_C) \right], \end{aligned} \quad (6.6)$$

where $\gamma = 0.57721 \dots$ is Euler's constant, $r_C = 1/m_N \alpha = 28.82 \text{ fm}$ with $\alpha = 1/137.036$ is the fine-structure constant, $\delta_{pp}^e(k)$ is the phase shift of low-energy elastic pp scattering in terms of the S-wave scattering length $a_{pp}^e = (-7.8196 \pm 0.0026) \text{ fm}$ and the effective range $r_{pp}^e = (2.790 \pm 0.014) \text{ fm}$ [30]. At $\alpha \rightarrow 0$ and $k \rightarrow 0$ the wave function $\psi_{pp}(k)$ tends to unity $\psi_{pp}(k) \rightarrow 1$ (see [40] and Appendix A).

The theoretical cross section for the reaction $pp \rightarrow pp\pi^0$ together with experimental data are depicted in Fig.5. In Table 1 we adduce numerical values of the theoretical and experimental cross section for the reaction $pp \rightarrow pp\pi^0$.

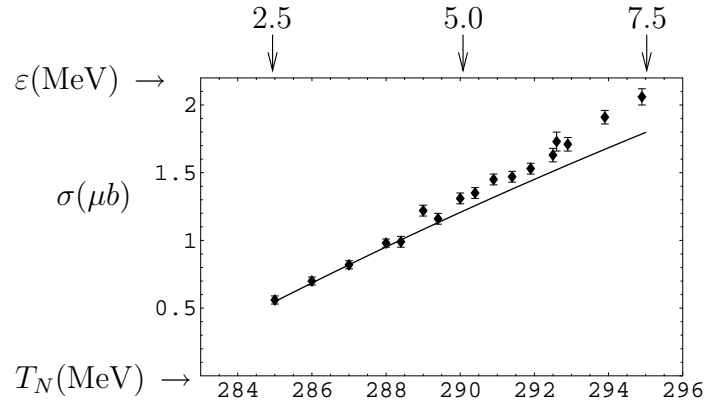


Figure 5: The theoretical cross-section for the reaction $pp \rightarrow pp\pi^0$, defined by the ${}^3P_0 \rightarrow {}^1S_0$ transition. The experimental data are taken from [36], T_N is a kinetic energy of the incident proton in the laboratory frame.

$T_N(\text{MeV})$	285	286	287	288	289	290	291	292
$\sigma^{(\text{exp})}(T_N)$	0.56(3)	0.70(3)	0.82(3)	0.98(3)	1.22(4)	1.31(4)	1.45(4)	1.53(4)
$\sigma^{(\text{th})}(T_N)$	0.55	0.69	0.82	0.95	1.08	1.21	1.33	1.45

Table 1: The theoretical and experimental values of the cross section for the reaction $pp \rightarrow pp\pi^0$, caused by the ${}^3P \rightarrow {}^1S_0$ transition. The experimental data are taken from [36]. The cross section is measured in μb .

The theoretical cross section agrees well with the experimental data. This implies that the analysis of the strangeness production near threshold of pN collisions given above works also well for the description of the pion-production in the pp collisions. The cross sections for other channels $pN \rightarrow NN\pi$ [39] can be calculated in a similar way.

7 Cross section for the reaction $\pi^0 p \rightarrow K^+ \Lambda^0$

Within chiral Lagrangian approach the reaction $\pi^0 p \rightarrow K^+ \Lambda^0$ has been investigated in [29] (see also [15]). Due to isospin invariance the amplitude of the reaction $\pi^- p \rightarrow K^0 \Lambda^0$ is related to the amplitude of the reaction $\pi^0 p \rightarrow K^+ \Lambda^0$ as

$$M(\pi^- p \rightarrow K^0 \Lambda^0) = \sqrt{2} M(\pi^0 p \rightarrow K^+ \Lambda^0). \quad (7.1)$$

The relative threshold momentum of the $\pi^0 p$ pair is equal to $p_0 = 520$ MeV. At threshold we define the amplitude of the reaction $\pi^0 p \rightarrow K^+ \Lambda^0$

$$M(\pi^0 p \rightarrow K^+ \Lambda^0) = 8\pi(m_K + m_{\Lambda^0}) a_{K^+ \Lambda^0}(p_0) \quad (7.2)$$

and the cross section [28]

$$\sigma^{\pi^0 p \rightarrow K^+ \Lambda^0}(p_{\pi^0}) = A_{K^+ \Lambda^0} p_{K^+ \Lambda^0}. \quad (7.3)$$

Here $p_{K^+ \Lambda^0}$ is a relative momentum of the $K^+ \Lambda^0$ pair, measured in GeV, and $A_{K^+ \Lambda^0}$ is equal to

$$A_{K^+ \Lambda^0} = \frac{4\pi}{p_0} |a_{K^+ \Lambda^0}(p_0)|^2. \quad (7.4)$$

For the amplitude $a_{K^+ \Lambda^0}$ we obtain the following expression

$$\begin{aligned} a_{K^+ \Lambda^0}(p_0) &= \frac{\sqrt{m_{\Lambda^0} m_N}}{4\pi(m_K + m_{\Lambda^0})} \left[\frac{1}{\sqrt{3}} (3 - 2\alpha_D) \left(\frac{g_{\pi NN}^2}{m_N + m_K + m_{\Lambda^0}} - \frac{g_{\pi NN}^2}{g_A^2} \frac{1}{2m_N} \right) \right. \\ &\quad - \frac{2}{\sqrt{3}} \alpha_D (2\alpha_D - 1) \frac{g_{\pi NN}^2 (m_{\Sigma^0} + m_K - m_N)}{m_{\Sigma^0}^2 + 2m_K T_N - (m_N - m_K)^2} - \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_{D1}) g_{\pi NN_1}^2}{m_{N_1} - m_K - m_{\Lambda^0}} \\ &\quad \left. - \frac{1}{\sqrt{3}} \frac{(3 - 2\alpha_{D2}) g_{\pi NN_2}^2}{m_{N_2} - m_K - m_{\Lambda^0}} + \frac{2}{\sqrt{3}} \alpha_{D2} (2\alpha_{D2} - 1) \frac{g_{\pi NN_2}^2 (m_{\Sigma_2^0} + m_N - m_K)}{m_{\Sigma_2^0}^2 + 2m_K T_N - (m_N - m_K)^2} \right] = \\ &= (0.109 - 0.190_{\Sigma^0} + 0.073_{N_1} - 0.047_{N_2} + 0.005_{\Sigma_2}) \text{ fm} = -0.050 \text{ fm}, \end{aligned} \quad (7.5)$$

where $m_{\Sigma_2} = 1750$ MeV. We remind that the resonance $\Sigma(1750)$ is the octet partner of the $N(1650)$ -resonance [7]. One can show that the contribution of the resonance $\Sigma(1620)$, the octet partner of the resonance $N(1535)$ [7], is negligible small.

For the constant $A_{K^+ \Lambda^0}$ we get the value

$$A_{K^+ \Lambda^0} = 0.61 \text{ m b/GeV}. \quad (7.6)$$

Due to isospin invariance the constant $A_{K^0 \Lambda^0}$ of the reaction $\pi^- p \rightarrow K^0 \Lambda^0$ is equal to

$$A_{K^0 \Lambda^0} = 2A_{K^+ \Lambda^0} = 1.22 \text{ m b/GeV}. \quad (7.7)$$

Near threshold of the reaction $\pi^- p \rightarrow K^0 \Lambda^0$ the constant $A_{K^0 \Lambda^0}$ defines the cross section [28]

$$\sigma^{\pi^- p \rightarrow K^0 \Lambda^0}(p_{\ell ab}) = A_{K^0 \Lambda^0} p_{K^0 \Lambda^0}, \quad (7.8)$$

where $p_{\ell ab}$ is a π^- -meson momentum in the laboratory frame and $p_{K^0 \Lambda^0}$ is the relative momentum of the $K^0 \Lambda^0$ pair. The theoretical value $A_{K^+ \Lambda^0} = 1.22 \text{ m b/GeV}$ agrees well with the experimental data $A_{K^0 \Lambda^0}^{(\text{exp})} = (1.23 \pm 0.23) \text{ m b/GeV}$ [28]. The cross section for the reaction $\pi^- p \rightarrow K^0 \Lambda^0$ Eq.(7.8) agrees also well with the theoretical results obtained in [29] within $SU(3)$ chiral dynamics with coupled channels.

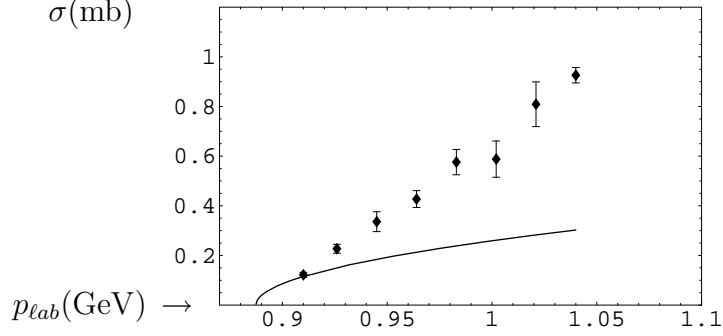


Figure 6: The theoretical cross-section for the reaction $\pi^- p \rightarrow \Lambda^0 K^0$ with the $\pi^- p$ and $\Lambda^0 K^0$ pairs in the S-wave state. The experimental points are taken from [28], p_{lab} is a momentum of the incident π^- -meson in the laboratory frame.

In the laboratory frame the threshold momentum of the π^- -meson is equal to $p_{th} = 0.887$ GeV. In the vicinity of threshold the momentum $p_{K^0 \Lambda^0}$ of the $K^0 \Lambda^0$ pair depends on the laboratory momentum p_{lab} of the π^- -meson as follows

$$p_{K^0 \Lambda^0} = \sqrt{\frac{2m_p^2 m_{\Lambda^0} m_K (p_{lab}^2 - p_{th}^2)}{(m_{\Lambda^0} + m_K)^2 ((m_{\Lambda^0} + m_K)^2 - m_p^2 - m_\pi^2)}} = 0.472 \sqrt{p_{lab}^2 - p_{th}^2}. \quad (7.9)$$

It is measured in GeV. For the cross section for the reaction $\pi^- p \rightarrow K^0 \Lambda^0$ as a function of p_{lab} we get

$$\sigma^{\pi^- p \rightarrow K^0 \Lambda^0}(p_{lab}) = (0.60 \pm 0.11) \sqrt{p_{lab}^2 - p_{th}^2} \quad (7.10)$$

For $p_{lab} = 0.910$ GeV the theoretical cross section $\sigma^{\pi^- p \rightarrow K^0 \Lambda^0}(p_{lab}) = 0.117$ mb agrees well with the experimental one $\sigma_{exp}^{\pi^- p \rightarrow K^0 \Lambda^0}(p_{lab}) = 0.122 \pm 0.010$ mb [28].

One can see that unlike the reactions $pN \rightarrow N\Lambda^0 K^+$ the contribution of the resonances $N(1535)$ and $N(1650)$ is very important for the correct description of the reactions $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$ near threshold of the final states [29]. Indeed, the neglect of the resonance contributions leads to the increase of the cross section for the reaction $\pi^- p \rightarrow \Lambda^0 K^+$ by a factor 3. This disagrees with the experimental data [28]. The importance of the contributions of the resonances $N(1535)$ and $N(1650)$ is caused by the substantial cancellation of the non-resonance contributions described by the first two terms in r.h.s. of Eq.(7.5).

In Fig. 6 we show that the region of the applicability of our theoretical cross section is restricted from above by the laboratory momentum $p_{lab} \leq 0.910$ GeV. This corresponds to $0 \leq p_{K^0 \Lambda^0} \leq 0.095$ GeV. In terms of the energy excess this region is defined by $0 \leq \varepsilon \leq 13$ MeV, where $\varepsilon = p_{K^0 \Lambda^0}^2 / 2\mu$ and $\mu = m_K m_{\Lambda^0} / (m_K + m_{\Lambda^0})$ is the reduced mass of the $K^0 \Lambda^0$ pair. It is obvious that the curve in Fig. 6 is a branch of hyperbola.

8 Conclusion

Using chiral Lagrangians with linear realisation of chiral $SU(3) \times SU(3)$ symmetry and non-derivative meson-baryon couplings we have calculated the cross sections for the reactions $pN \rightarrow N\Lambda^0 K^+$ near threshold of the final $N\Lambda^0 K^+$ state. We have taken into

account the contributions of the rescattering in the initial pN state and the final-state interactions in the $N\Lambda^0$ channels. We have shown that the ratio of the cross sections is equal to $R \simeq 3$. This agrees with an enhancement of the strangeness production in the pn interaction with respect to the strangeness production in the pp interaction observed by the ANKE-Collaboration at COSY [1]. We have explained such an enhancement by the contribution of the pn interaction in the isospin-singlet state, which is much stronger than the pn interaction in the isospin-triplet state.

We have shown that the contribution of the vector-meson exchanges is at the level of a few percent. Hence, the vector-meson exchanges cannot be fully responsible for the dynamics of strong low-energy pN interactions near threshold of the reaction $pN \rightarrow N\Lambda^0 K^+$. We have also shown that the contribution of the resonances $N(1535)$ and $N(1650)$, treated as elementary particles, to the matrix elements of the $pN \rightarrow N\Lambda^0 K^+$ transitions does not exceed 8%. However, such an assertion should not be valid within Wilkin's approach [6, 8], where the main contributions to the amplitudes of the reactions $pN \rightarrow N\Lambda^0 K^+$ come from the resonances $N(1535)$ and $N(1650)$, described by the Breit-Wigner shapes.

In addition to vector-meson exchanges and resonances $N(1535)$ and $N(1650)$ one can estimate the contributions of the scalar meson $f_0(980)$ and the tensor meson $f_2(1270)$ [7]. Following [42] and [43, 44] one finds that the contribution of the scalar meson $f_0(980)$ makes up about 2% [42], whereas the contribution of the tensor meson $f_2(1270)$ is about 5.7% for the coupling constants $g_{f_2 NN}^{(1)} \simeq 2$ and $g_{f_2 NN}^{(2)} = 0$ [43, 44]. Since the theoretical accuracy of our approach is not better than 15%, one can drop the contributions of vector, tensor, exotic scalar mesons and resonances $N(1535)$ and $N(1650)$ with respect to the contributions of ground state baryon octet coupled to octets of pseudoscalar mesons and their chiral scalar partners, which are taken in the infinite mass limit.

For the confirmation of our results, obtained for the strangeness production near threshold of pN interactions, we have calculated the cross sections for the reactions $pp \rightarrow pp\pi^0$, $\pi^0 p \rightarrow \Lambda^0 K^+$ and $\pi^- p \rightarrow \Lambda^0 K^0$. The theoretical results agree well with the experimental data. Unlike the strangeness production in the pN interactions the contribution of resonances $N(1535)$ and $N(1650)$ to the amplitudes of the reactions $\pi N \rightarrow \Lambda^0 K$ has turned out to be very important. This is caused by the substantial cancellation of the non-resonance contributions, defined by interactions of ground state baryon octet with octets of pseudoscalar and scalar mesons.

9 Acknowledgement

We are grateful to Vladimir Koptev for calling our attention to the problem, investigated in this paper, and numerous and fruitful discussions stimulating this work.

One of the authors (A. Ivanov) is grateful to Colin Wilkin, Alexander Sibirtsev, Alexander Kudryavtsev, Vadim Baru and all participants of the seminar at Institut für Kernphysik at Forschungszentrum in Jülich for fruitful discussions and constructive critical comments. A. Ivanov thanks Prof. Ströher for warm hospitality extended to him during his stay at Institut für Kernphysik at Forschungszentrum in Jülich. The assistance of Yurii Valdaу during the stay of A. Ivanov at Institut für Kernphysik is greatly appreciated.

We are grateful to Igor Strakovsky for the discussions and comments on the SAID analysis of the experimental data on pp scattering [32, 33].

Appendix A: Cross sections for the reaction $pN \rightarrow N\Lambda^0 K^+$ accounting for the final-state interaction

The amplitude of the reaction $pN \rightarrow N\Lambda^0 K^+$, where $N\Lambda^0$ pair in the S-wave state, is related to the matrix element of the \mathbb{T} -matrix as follows

$$M(pN \rightarrow N\Lambda^0 K^+) = \lim_{TV \rightarrow \infty} \frac{\langle N\Lambda^0 K^+ | \mathbb{T} | pN \rangle}{VT}, \quad (\text{A-1})$$

where TV is a 4-dimensional volume defined by $(2\pi)^4 \delta^{(4)}(0) = TV$ and the \mathbb{T} -matrix obeys the unitary condition [45]

$$\mathbb{T} - \mathbb{T}^\dagger = i \mathbb{T}^\dagger \mathbb{T}. \quad (\text{A-2})$$

The amplitude $M(pN \rightarrow N\Lambda^0 K^+)$, defined by Eq.(A-1), describes the total amplitude of the reaction $pN \rightarrow N\Lambda^0 K^+$ accounting for the all kinds of interactions.

In order to separate the contributions of the final-state interaction we propose to follow [46] (see also [17] and [40]). First we define the amplitude of the reaction $pN \rightarrow N\Lambda^0 K^+$, which is calculated by the diagrams depicted in Fig.1–Fig.4:

$$M_0(pN \rightarrow N\Lambda^0 K^+) = \lim_{TV \rightarrow \infty} \frac{\langle N\Lambda^0 K^+(0) | \mathbb{T} | pN \rangle}{VT}, \quad (\text{A-3})$$

where $|N\Lambda^0 K^+(0)\rangle$ is the wave function of the final state with free particles [46]. The account for the final-state interaction in the $N\Lambda^0$ channel with the $N\Lambda^0$ pair in the S-wave state can be given by means of the wave function $|N\Lambda^0 K^+(k_{N\Lambda})\rangle$ defined by [46]

$$|N\Lambda^0 K^+(k_{N\Lambda})\rangle = e^{i\delta_{N\Lambda^0}(k_{N\Lambda^0})} \frac{\sin \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}(-a_{N\Lambda^0})} |N\Lambda^0 K^+(0)\rangle, \quad (\text{A-4})$$

where $k_{N\Lambda^0}$ is a relative moment of the $N\Lambda^0$ pair, $\delta_{N\Lambda^0}(k_{N\Lambda^0})$ and $a_{N\Lambda^0}$ are the phase shift and the S-wave scattering length of $N\Lambda^0$ scattering in the S-wave state. As has been shown in [19] (see also [40]) such a wave function (A-4) can be obtained by summing up an infinite series of bubble-diagrams in the $N\Lambda^0$ channel.

The wave function $|N\Lambda^0 K^+(0)\rangle$ is normalised by [19]

$$\begin{aligned} {}'\langle K^+(0)\Lambda^0 N | N\Lambda^0 K^+(0)\rangle &= (2\pi)^3 2E_{K^+}(\vec{p}_{K^+}) \delta^{(3)}(\vec{p}_{K^+}' - \vec{p}_{K^+}) \\ &\times (2\pi)^3 2E_{\Lambda^0}(\vec{p}_{\Lambda^0}) \delta^{(3)}(\vec{p}_{\Lambda^0}' - \vec{p}_{\Lambda^0}) \delta_{\sigma_{\Lambda^0}' \sigma_{\Lambda^0}} (2\pi)^3 2E_N(\vec{p}_N) \delta^{(3)}(\vec{p}_N' - \vec{p}_N) \delta_{\sigma_N' \sigma_N}, \end{aligned} \quad (\text{A-5})$$

where \vec{p}_{K^+}, \vec{p}_X and $\vec{p}_{K^+}', \vec{p}_X'$ with $X = \Lambda^0, N$ are momenta of particles in the initial and final states, $E_{K^+}(\vec{p}_{K^+})$ is the K^+ -meson energy, $E_X(\vec{p}_X)$ and σ_X are the energy and polarisation of the particle X . Due to (A-5) the wave function $|N\Lambda^0 K^+(k_{N\Lambda})\rangle$ is normalised as follows

$$\begin{aligned} {}'\langle K^+(k_{N\Lambda^0})\Lambda^0 N | N\Lambda^0 K^+(k_{N\Lambda^0})\rangle &= \frac{\sin^2 \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}^2 a_{N\Lambda^0}^2} (2\pi)^3 2E_{K^+}(\vec{p}_{K^+}) \delta^{(3)}(\vec{p}_{K^+}' - \vec{p}_{K^+}) \\ &\times (2\pi)^3 2E_{\Lambda^0}(\vec{p}_{\Lambda^0}) \delta^{(3)}(\vec{p}_{\Lambda^0}' - \vec{p}_{\Lambda^0}) \delta_{\sigma_{\Lambda^0}' \sigma_{\Lambda^0}} (2\pi)^3 2E_N(\vec{p}_N) \delta^{(3)}(\vec{p}_N' - \vec{p}_N) \delta_{\sigma_N' \sigma_N} = P(k_{N\Lambda^0}) \\ &\times (2\pi)^3 2E_{\Lambda^0}(\vec{p}_{\Lambda^0}) \delta^{(3)}(\vec{p}_{\Lambda^0}' - \vec{p}_{\Lambda^0}) \delta_{\sigma_{\Lambda^0}' \sigma_{\Lambda^0}} (2\pi)^3 2E_N(\vec{p}_N) \delta^{(3)}(\vec{p}_N' - \vec{p}_N) \delta_{\sigma_N' \sigma_N}. \end{aligned} \quad (\text{A-6})$$

The factor $P(k_{N\Lambda^0})$ can be rewritten in terms of the cross sections for $N\Lambda^0$ scattering in the S-wave state

$$P(k_{N\Lambda^0}) = \frac{\sin^2 \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}^2 a_{N\Lambda^0}^2} = \frac{\sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(k_{N\Lambda^0})}{\sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(0)} \leq 1, \quad (\text{A-7})$$

where the cross sections $\sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(k_{N\Lambda^0})$ and $\sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(0)$ are equal to

$$\sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(k_{N\Lambda^0}) = 4\pi \frac{\sin^2 \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}^2}, \quad \sigma^{N\Lambda^0 \rightarrow N\Lambda^0}(0) = 4\pi a_{N\Lambda^0}^2. \quad (\text{A-8})$$

The factor $P(k_{N\Lambda^0})$ can be interpreted as a probability to find a non-interacting system $N\Lambda^0 K^+$ with a relative momentum $k_{N\Lambda^0}$ of the $N\Lambda^0$ pair [45]. For $k_{N\Lambda^0} = 0$, when the particles N and Λ^0 are separated by an infinite relative distance $r_{N\Lambda^0} \sim 1/k_{N\Lambda^0} \rightarrow \infty$ at $k_{N\Lambda^0} \rightarrow 0$, the probability to find non-interacting particles in the state $|N\Lambda^0 K^+\rangle$ is equal to unity, i.e. $P(k_{N\Lambda^0} = 0) = 1$. For any finite $k_{N\Lambda^0}$ it is less than unity, i.e. $P(k_{N\Lambda^0}) \leq 1$ [45].

The amplitude of the reaction $pN \rightarrow N\Lambda^0 K^+$ accounting for the final-state interaction in the $N\Lambda^0$ channel is defined by [46]

$$\begin{aligned} M(pN \rightarrow N\Lambda^0 K^+) &= \lim_{TV \rightarrow \infty} \frac{\langle N\Lambda^0 K^+(k_{N\Lambda^0}) | \mathbb{T} | pN \rangle}{VT} = \\ &= \lim_{TV \rightarrow \infty} e^{-i\delta_{N\Lambda^0}(k_{N\Lambda^0})} \frac{\sin \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}(-a_{N\Lambda^0})} \frac{\langle N\Lambda^0 K^+(0) | \mathbb{T} | pN \rangle}{VT} = \\ &= e^{-i\delta_{N\Lambda^0}(k_{N\Lambda^0})} \frac{\sin \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}(-a_{N\Lambda^0})} M_0(pN \rightarrow N\Lambda^0 K^+). \end{aligned} \quad (\text{A-9})$$

The same result can be obtained by summing up an infinite series of bubble diagrams in the $N\Lambda^0$ channel [19] (see also [40]). Hence the amplitude of the reaction $pN \rightarrow N\Lambda^0 K^+$ taking into account the final-state interaction in the $N\Lambda^0$ channel is

$$M(pN \rightarrow N\Lambda^0 K^+) = e^{-i\delta_{N\Lambda^0}(k_{N\Lambda^0})} \frac{\sin \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}(-a_{N\Lambda^0})} M_0(pN \rightarrow N\Lambda^0 K^+), \quad (\text{A-10})$$

where the amplitude $M_0(pN \rightarrow N\Lambda^0 K^+)$ is defined by the Feynman diagrams in Fig.1–Fig.4.

At $k_{N\Lambda^0} = 0$ we get $M(pN \rightarrow N\Lambda^0 K^+) = M_0(pN \rightarrow N\Lambda^0 K^+)$. The reduction of the amplitude of the final-state $N\Lambda^0$ interaction to unity at $k_{N\Lambda^0} = 0$ is clear due to the short-range character of strong interactions. Indeed, at $k_{N\Lambda^0} = 0$, corresponding to infinitely large relative distance of the baryons in the $N\Lambda^0$ pair, the baryons do not influence each other, since the wave functions of them do not overlap.

The contribution of the Coulomb interaction can be taken into account according to the prescription [46] and Balewski *et al.* [17, 20]. This gives

$$M(pN \rightarrow N\Lambda^0 K^+) = e^{-i\delta_{N\Lambda^0}(k_{N\Lambda^0})} \frac{\sin \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}(-a_{N\Lambda^0})} C_0(k_{NK^+}) M_0(pN \rightarrow N\Lambda^0 K^+). \quad (\text{A-11})$$

Here $C_0(k_{NK^+})$ is the Gamow factor [30, 46]

$$C_0^2(k_{NK^+}) = \frac{2\pi\alpha Z_N \eta}{e^{2\pi\alpha Z_N \eta} - 1}, \quad (\text{A-12})$$

where $\alpha = 1/137.036$ is the fine-structure constant, k_{NK^+} is a relative momentum of the NK^+ pair, $\eta = \mu_{NK^+}/k_{NK^+} = 1/v_{NK^+}$ is the inverse relative velocity of the NK^+ pair, $Z_N = 1$ for $N = p$ and $Z_N = 0$ for $N = n$.

In the case of the Coulomb interaction the wave function $|N\Lambda^0 K^+\rangle$ has the normalisation (A-6) with the factor $P(k_{N\Lambda^0}, k_{NK^+})$ equal to

$$P(k_{N\Lambda^0}, k_{NK^+}) = C_0^2(k_{NK^+}) \frac{\sin^2 \delta_{N\Lambda^0}(k_{N\Lambda^0})}{k_{N\Lambda^0}^2 a_{N\Lambda^0}^2} \leq 1. \quad (\text{A-13})$$

It defines the probability to find a non-interacting system $N\Lambda^0 K^+$ at relative momenta $k_{N\Lambda^0}$ and k_{NK^+} .

The cross section for the reaction $pN \rightarrow N\Lambda^0 K^+$ with the account for the final-state interaction is defined by

$$\sigma^{pN \rightarrow N\Lambda^0 K^+}(\varepsilon) = A_{pN} \varepsilon^2 \Omega_{N\Lambda^0 K^+}(\varepsilon), \quad (\text{A-14})$$

where A_{pN} is a constant, calculated at threshold momentum.

The definition (A-14) is correct in the vicinity of threshold, where for the calculation of the amplitude $M_0(pN \rightarrow N\Lambda^0 K^+)$ of the reaction $pN \rightarrow N\Lambda^0 K^+$ (or the constant A_{pN}) one can neglect the dependence on the momenta of the particle in the final state $N\Lambda^0 K^+$. The function $\Omega_{N\Lambda^0 K^+}(\varepsilon)$ is determined by

$$\Omega_{N\Lambda^0 K^+}(\varepsilon) = \frac{2}{\pi^3} \int f_{\text{FSI}}(\varepsilon, x) \delta(1 - \vec{x}^2 - \vec{y}^2) C_0^2(\vec{x}, \vec{y}) d^3x d^3y, \quad (\text{A-15})$$

where the function $f_{\text{FSI}}(\varepsilon, x)$ describes the final-state interaction in the $N\Lambda^0$ channel [17]. It is normalised to unity at $\varepsilon = 0$ and equal to [17, 18]

$$f_{\text{FSI}}(\varepsilon, x) = \frac{1}{\left(1 - \frac{\varepsilon m_{\Lambda^0} m_N}{m_{\Lambda^0} + m_N} a_{N\Lambda^0} r_{N\Lambda^0} x^2\right)^2 + \frac{2\varepsilon m_{\Lambda^0} m_N}{m_{\Lambda^0} + m_N} a_{N\Lambda^0}^2 x^2}, \quad (\text{A-16})$$

where $a_{N\Lambda^0} = -2 \text{ fm}$ and $r_{N\Lambda^0} = 1 \text{ fm}$ are the S-wave scattering length and effective range of $N\Lambda^0$ scattering [17].

The Gamow factor depends on the variables \vec{x} and \vec{y} as follows

$$\eta = \sqrt{\frac{m_p(m_{\Lambda^0} + m_p)}{2\varepsilon m_{\Lambda^0}}} \frac{1}{\left|\vec{x} - \sqrt{\frac{m_p(m_{\Lambda^0} + m_p + m_K)}{m_{\Lambda^0} m_K}} \vec{y}\right|} = \frac{1}{v} \sqrt{\frac{m_p(m_{\Lambda^0} + m_p)}{2\varepsilon m_{\Lambda^0}}}. \quad (\text{A-17})$$

The function $\Omega_{N\Lambda^0 K^+}(\varepsilon)$ is equal to to unity for $\alpha = 0$ and $a_{N\Lambda^0} = r_{N\Lambda^0} = 0$. For the reactions $pp \rightarrow p\Lambda^0 K^+$ and $pn \rightarrow n\Lambda^0 K^+$ we get

$$\Omega_{p\Lambda^0 K^+}(\varepsilon) = \sqrt{\frac{m_K(m_{\Lambda^0} + m_p)}{2\varepsilon(m_{\Lambda^0} + m_p + m_K)}} \frac{8}{\pi} \int_0^1 \int_{v-(x)}^{v+(x)} \frac{f_{\text{FSI}}(\varepsilon, x) 2\pi\alpha dv dx}{\exp\left\{\frac{2\pi\alpha}{v} \sqrt{\frac{m_p(m_{\Lambda^0} + m_p)}{2\varepsilon m_{\Lambda^0}}}\right\} - 1}, \quad (\text{A-18})$$

where we have denoted

$$\begin{aligned} v_+ &= x + \sqrt{\frac{m_p(m_{\Lambda^0} + m_p + m_K)}{m_{\Lambda^0}m_K}} \sqrt{1-x^2}, \\ v_- &= \left| x - \sqrt{\frac{m_p(m_{\Lambda^0} + m_p + m_K)}{m_{\Lambda^0}m_K}} \sqrt{1-x^2} \right| \end{aligned} \quad (\text{A-19})$$

and

$$\Omega_{n\Lambda^0 K^+}(\varepsilon) = \frac{16}{\pi} \int_0^1 f_{\text{FSI}}(\varepsilon, x) x^2 \sqrt{1-x^2} dx. \quad (\text{A-20})$$

For $\varepsilon = 6.68$ MeV the contribution of the Coulomb repulsion in the K^+p pair to function $\Omega_{p\Lambda^0 K^+}(\varepsilon)$ makes up about 15%. At the neglect of the contribution of the Coulomb repulsion $\Omega_{p\Lambda^0 K^+}(\varepsilon) = \Omega_{n\Lambda^0 K^+}(\varepsilon) = 0.67$ at $\varepsilon = 6.68$ MeV.

In the reaction $pp \rightarrow pp\pi^0$ the account for the final-state interaction in the pp channel has been carried out within the scheme expounded above. As we have shown the theoretical cross sections for the reactions $pp \rightarrow p\Lambda^0 K^+$ and $pp \rightarrow pp\pi^0$ agree well with the experimental data.

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